Sinthôme:

mutant automata in an ill-founded world

A Dissertation Submitted to the
Division of Media and Communications
of the European Graduate School
in Candidacy for the Degree of
Doctor of Philosophy

By Terrence HANDSCOMB

August 2011

Dissertation Defended August 6, 2011, Saas-Fee, Switzerland

Awarded Summa Cum Laude

Doctoral Defense Committee:

Professor Alain Badiou (René Descartes Chair, European Graduate School; Professor Emeritus, École Normale Supérieure, Paris, France)

Professor Laurence A. Rickels (Sigmund Freud Chair, European Graduate School; Professor, Staatliche Akademie der Bildenden Künste, Karlsruhe, Germany)

Professor Martin Hielscher (Theodor W. Adorno Chair and Dean, Media and Communications Division, European Graduate School; Professor, Ludwig-Maximilians-Universität, München, Germany)

Abstract

Sinthôme:

mutant automata in an ill-founded world

by

Terrence Handscomb

Alain Badiou's recent discussions of a subjectivizable body is typically expressed in his unique intertextual application of mathematics and philosophy. In *Logics of Worlds* (Badiou 2009) the notions of worlds, events, traces and a "subjectivizable body," involve a dialectic synthesis of being and appearance. This synthesis is formally grounded in a functor category of sheaves over a Grothendieck topos (site). Summoning some of Badiou's key themes – subject, site, event, body, trace and world – the theory of a subject-body in the form an abstract mutant automaton, is proposed.

An abstract mutant automaton can be mathematically described as a dynamic final state machine with transitions between unknown state spaces. These states and transitions can be schematized as final coalgebras with terminal objects and a smallest dynamic function
'**`go-on`**.' The mutable dynamics conclude when an exception is identified and named. Terminal coalgebras are the mathematical duals of initial algebras. Badiou's 'transcendental' ordering of a world is an initial algebra. In contradistinction, mutant automata are final coalgebras, which range across *cotranscendental* space – the logic of any evental trace is necessarily cotranscendental.

Badiou's formal description of a site and the formation of a subjectivizable body involves the aporetic anomaly of non-well-founded sets. Sites are construed as the proper class of ontological singularities. A protean-automaton-subject is spawned of a site, born of the epiphany of a *silent singularity* and precipitates under the mutable trace of the *sinthôme*.

Unlike Badiou's faithful subject-body, mutant automata cannot be dialectically sublated. Nor can their dissembled body be retroactively "… gathered under the trace of the vanishing event." (299) Mutant automata are not the faithful subjects of events. Following the occurrence of a pseudo-dialectic turn of events, mutant-automaton-subjects instead suffuse the registers of the imaginary at the level of the symptom.

She walked back to the car and sat for a long while in the parking lot, idling the engine and watching a woman in a muumuu walk out of the Carolina Pines Motel and cross the street to a supermarket. The woman walked in small mincing steps and kept raising her hand to shield her eyes from the vacant sunlight. As if in a trance Maria watched the woman, for it seemed to her then that she was watching the dead still center of the world, the quintessential intersection of nothing.

Joan Didion, *Play It As It Lays: a novel* (Didion 1970, 66-67)

Table of Contents

Introduction

Proem

The relationship between mathematics, literature and philosophy has been discussed in a number of theoretical texts. A number of these texts focus on the relationship between mathematics and philosophy in the writings of Alain Badiou (Ackerley 1998; Duffy 2006; Gillespie 2008; Widder 2001). Mathematics plays a significant role of Badiou's *oeuvre* and is a powerful force in the intertextual discursiveness that characterizes his philosophical writings. However, Badiou's mathematics entails a hybrid use of set theories, category theories and algebras that involve arrays of abstract signs, rules, relationships and interpretations, not all of which are 'mathematical.' Badiou's writings are further complicated by the philosophical burden he places on set-theoretical mathematics. For example: "… Badiou's reader must accept a tautological equivalence between being and multiplicity in order to accept the assertion that mathematics is ontology … " (Gillespie 2008, 25). Badiou's earlier set-theoretical mathematics, while dominant in *Being and Event* (Badiou 2005), is further expanded in *Logics of Worlds* (Badiou 2009b) where the question of pure set-theoretical ontology is indexed to the partial orders of an existential algebra of appearing. Badiou's algebraic theory of the evental appearing of pure being is itself localized in the topology of a site. A site is schematically equivalent to what is known in mathematics as the functor category of sheaves over a topological space, specifically a Grothendieck topos. Unlike the ontology of discrete set-theoretical multiples found in *Being and Event*, a site is a smooth topology of open sets and covers.

Badiou is quite aware of the impact his writing have on both professional mathematicians and those readers less proficient in mathematics. The former tend to dismiss Badiou's 'mathematics' as being too metaphorical, while the latter tend to gloss over his extensive mathematical detail and focus instead on the entailed philosophy. At the beginning of a 1977 essay on mathematical induction and torsion Badiou is unapologetic:

> I would like to talk about a certain use of mathematics that is properly my own without seeming proper to anyone else: neither to mathematicians who find it too metaphorical, nor to others, who are intimidated by it. (Badiou 2009c, 148).

To readers already familiar with Zermelo-Fraenkel set theory, category theory, Cohen forcing, Grothendieck topology and Heyting algebras, the challenge may not be harassing. However, to many established career mathematicians who find Badiou's 'certain use' of mathematics to be metaphorical, often fail to see how Badiou plays to an audience well aware of the allegorical, rather than a figurative efficacy of his mathematics. Badiou's mathematics exhibit the theatricality of a Lacanian matheme and the authenticity of an ardent philosopher. In a professional mathematical sense, Badiou's mathematics is undeniably non-normal: this is also true of my own 'certain use' of mathematics.

In this dissertation, my exegesis of Badiou's mathematics is implacable and I often use my own notation and interpretive nomenclature that cannot be found in Badiou's texts. My mathematical symbolism is usually closer to the mathematical literature than it is to Badiou's denotational eccentricities. For example, I often interchange the terms "transcendental order" and "partially ordered set," where there is no ambiguity. My discussion of Cohen forcing in Chapter One, focuses on specific partial orders which are atomless Boolean algebras. In this interpretation of Cohen forcing, discrete multiples are ordered as classical algebraic intervals on the set of natural numbers. These Boolean algebras are not the same as the Heyting algebras that badiou uses to describe the 'transcendental order' of a world, both philosophically and mathematically. The ‘transcendental’ of a world, as it is described in *Logics of Worlds*, is a non-discrete real-line interval of coefficients of intensity, closed under maximal and minimal limits. On the other hand, my “*B*-transcendental” order of a situation is ontologically tied to the discrete finite natural numbers and their transfinite cardinal corelates.

With the radical transgressive exception of coalgebra, coinduction and bisimulation, my mathematics is not dissimilar to Badiou’s. However, my use of terminal coalgebras, coinduction and bisimulations may be new to readers familiar a Badiou’s work. Terminal coalgebras are the mathematical duals of initial algebras and they represent a radical reversal of Badiou’s algebraic notion of a transcendental order. In recent years, these mathematical forms have become widely discussed in the professional literature of theoretical computer science where they have risen to prominence.

Badiou is also well aware of the hermeneutic, ontological and existential potency of his program. His intertextual poïesis falls outside the authoritative scientism of mainstream mathematics. A full appreciation and understanding of Badiou’s mathematics involves much more than a purely formal understanding of abstract mathematical forms:

> Briefly put, it is a matter of short-circuiting the dialectical analysis by examining the way in which mathematics treats a word, so that without losing anything of its rigor, the treatment is nonetheless an interpretation of this word. … This is tantamount to saying that we consider the mathematical signifier a symptom around which the deductive text, without knowing this at all, attempts an auto-analysis. (ibid.)

In this passage the mathematical adjunction of word, sign and name are significant. Importantly, the passage gives *prima facie* ground and a viable *raison d’être* for the overarching theme of this dissertation: the coalgebraic development of the notion of a “*sinthôme*” and an abstract-mutant-automaton-subject (c.f. (Badiou 2009a, 455)) which occurs in imaginary registers on the level of the symptom. On the level of the symptom, the fastening of name to the trace follows the occurrence of what I shall call a *pseudo-dialectic turn*, which may precipitate the radical formation of a subject-body and its mutable affectation. However, this involves a reassessment of Badiou's recent theory of the event and reverses the post-evental transcendental order of a world into its cotranscendental dual. The pathology of a subjectivizable body can then be schematically abstracted as a *mutant-automaton*, coalgebraically conceived under the transgressive sign of the *sinthôme* and the transitional principle of going-on.

Beginnings

Between 1986 and 1991 I produced a number of large format oil-alkyd paintings on polyester architectural drafting film. Typically, these works juxtaposed large typographic mathematical symbols with broad areas of intense, highly gesticulated brush strokes. Between 1994 and 1997 I produced two significant computer-based installations. The electronic works coalesced psychoanalytical and mathematical theory to articulate the notion of a mutable pathological subject.

My two computer-based installations *Before Information there were The Machines* (1995) and *Space Invaders: black satire and the BBS* (1997) theoretically presupposed an abstract algebraic body but treated such a body as an allegory for the internet. The graphic-user-interface was structured like the visible surface of the human body. A user could "enter" the "body" by triggering interactive hotspots whose figuration took the form of external bodily organs. I referred to these entry points as "logical gates" or "frames." The idea of logical frames came from Kripke semantics in which a logical frame is a type of the partial order of possible worlds. Kripke semantics is often known as possible world semantics. While assembling the research material for this document I realized that the mathematical structure of Kripke frames, a device widely used by modal logicians in the 1970s and 1980s, are partial ordered structures that are equivalent to Badiou's notion of the "transcendental" ordering of a world. I also noticed that modal logics are coalgebraic (Cîrstea et al. 2009).

Although some of the mathematical typography of my earlier paintings remained in my later digital work, the place of the typographic object, which was conceptually dominant and highly visible in the paintings, had been entirely subsumed by the programming language used to produce the installations. The presentation of the sign, which visually dominated the paintings, was in the interactive installations entirely overwritten and buried deep in the workings of the machine. Compiled machine code had replaced the visible significance of the sign with an invisible transcoded analog. By transcoding bodily signs, the machine had replaced the *visible* subject by regulating its signification. The machine had become the subject, inasmuch as the subjectivizable letter of the GUI(Graphic User Interface) had become the invisible psychological placeholder of the symptom. Signifiers whose link to some discernible signified had been completely mediated by the circularity of the executable program. However, the graphic surface became a twofold state of representation in which the hidden machine code has a highly subjectivizable 'surface.' This surface becomes both the metaphorical and actual body of the work. Of course, the graphic surface remains evident, for without it there would be no "subjectivizable" body (c.f. (Badiou 2009a, 455)). Without a representational surface, the work would otherwise need to be represented on the level of abstract automation in which the subject would be reconfigured as some other body that would be completely removed from the visible surface. Recently I developed a coalgebraic analogy for the machine-subject of these earlier works: a transgressive mutant automaton.

There are a number of conceptual and theoretical problems with my earlier works. The misleading idea that the subject may be lost in the passage of meaning between the inner functionality of machine to the GUI is arbitrary, since machine code is a *faithful* representation of the executable program. The idea that a user may lose control over the "body" of the work – there are trajectories and dead ends in the user navigation of the work, which lead to visceral, horrendous and culturally offensive "things." The user may inadvertently navigate to dark places and has no way of extracting him or herself, but the machine never relinquishes "control."

Although a recasting of the ides that a GUI may be designed in such a way as it metonymically represents the inscribable surface of the abstract body, the notion that states of representation can be affectively order abstract surfaces informs this dissertation. The notion that abstract partial orders and abnormal occurrences can form subjectivizable surfaces, may find meaning in a close reading of Badiou's major texts.

(Co)Transcendental functors

Badiou's notion of a "mathematical" relationship between being and existence, is rooted in the abstract ground of algebraic geometry, in the particular topological spaces named after the German born mathematician Alexander Grothendieck (b. 1928). The relationship between topological ontology and an algebraic theory of existence was first raised in *Théorie du sujet* (Badiou 1982) and later fully extrapolated in *Logiques des mondes: l'être et événement, 2* (Badiou 2006). Grothendieck topoi form the principle mathematical templates from which Badiou derives the main formal mechanisms summoned in *Logics of Worlds*. The topological linking of pure ontological matter to the indices of existential appearance, recasts Badiou's theory of the event. In his earlier *L'être et l'événement* (Badiou 1988), the ontology of the event is theoretically grounded in set-theory and the aporetic properties of ontological singularities, such as those multiples which contain themselves as elements.

Badiou's conceptual indexical operation of linking of open-set topology to an algebra of appearance, lays in the categorial structure know as functors. Badiou retools basic functor operations into what he refers to as transcendental functors (Badiou 2009b, 277-295). Functor operations allow for the pointwise convergence of the existential category of appearance and the ontological category of smooth topologies. Mutable subjective existence can be momentarily stabilized by 'localizing' the ontological ground of being as an existential being-there. This can be achieved through the indexical actions of transcendental functors. Functors are category theoretic operations, which differ from set-theoretical functions inasmuch as functors range over objects and functions whereas functions range over objects in the category **Set**. [1] Badiou's schemata of an evental site are structurally equivalent to Grothendieck topoi: "A world, as a site of being-there, is a Grothendieck topos." (Badiou 2009a, 295)

Categories

Category theory is a relatively new branch of mathematics. Categories are mathematical entities that can be abstracted by formalizing them as collections of objects and the relationships these objects called arrows or morphisms. In this regard, one of the most accessible categories is the category **Set**. For example, Badiou's notion of the multiples that are presented in a situation can be thought of as the objects of the category **Set**. Importantly, category theory provides rich mathematical ground for the abstraction of the more general categories of ontology and existence as well as the formation of certain relationships between them. Functors are special categorial operations that can be formed not only over objects, but also over the operations themselves. Self-reference is a powerful categorial device, which is denied classical set theory.

The notion of a categorial arrow is abstracted from the structure of set-theoretical functions. Category theory effectively sidesteps a number of structural issues that arise in set theory: in particular the aporetic consequences of self-inclusion that form the ontological conditions of the event. Unlike set theory, category theory does not define properties of collections *internally* by the predicate of inclusion or set membership, but rather defines properties of collections *externally* by establishing relationships with other collections. Thus, a categorial arrow (or morphism), can be thought of as a particular kind

[1] Category theorists often denote categories with shortened abbreviations in a bold type face. We follow this convention.

relationship between two collections; a relationship that is established by the kind of arrow that is being used.

By specifying a certain kind of object and a certain kind of arrow, a category may also be thought of as a universe for a particular kind of discourse. For example, a categorial treatment of topology takes place in a world with topological spaces, such as abstract bodies as objects and continuous functions as arrows between them. Functors can also be thought of as 'subjectivizable' operations that can adopt a number of intertextual characteristics. When Badiou speaks of a condition whereby the " … topological disturbance of an algebra is the precise name for the fears whose roots lie in the extreme subjectivization of the splace … " [2] (Badiou 2009a, 260), he speaks of a subjectivizable function that is at play between two categories. The first is the category **Top** (the category of topological spaces), which is formed, in part, by the product category **Set** × **Exist**, while the second is the algebraic category I call **Exist**. The 'subjectivizing of the splace' can be thought of as an indexical functor object in the functor category ***F***↑: **Exist**→ **Set** × **Exist**. This functor can be thought of as an indexical operator which maps certain values from the existential category **Exist** to the onto-logical product category **Set** × **Exist**. Some of key functor operations which range over ontological and existential categories will prove to be crucial developmental tools in my coalgebraic discussion of the pseudo-dialectic turn and the *sinthôme*.

Cohen Forcing

[2] *Splace*: "…the legal place presupposed by any event." (Badiou 2009c, 76). In this fragment Badiou indicates that the compositional law of the place is algebraic. It is the law that must be presupposed if it is to be violated by a subject who is "subservient to the rule that determines a place, it [the subject] nevertheless punctuates the latter with the interruption of its effects" (ibid, 259). This action materially localizes the disruption precipitated by of the event.

In *Being and Event*, Badiou argues that set theory presents the only mode of discourse in which the ontological notion of pure being can be properly understood. By implication, the concepts of void, presentation, representation, situation, site and event can be philosophically developed from set-theoretic templates. For example, Badiou describes the notion of void as the empty set marked by the sign "$\varnothing$." In this configuration, $\varnothing$ marks the proper name of being (Badiou 2005, 52). In earlier discussions (i.e. before the publication of *Logics of Worlds*) the idea of an event and an anonymous subject, who is a finite configuration of a generic procedure "standing on the edge of the void," can only be understood in a finite set-theoretical system of representation that has been generically extended under the mathematical procedure known as Cohen forcing.

American mathematician Paul Cohen (1934–2007) developed mathematical procedures that demonstrated the independence of the continuum hypothesis (and the axiom of choice (AC)) from the axioms of Zermelo-Fraenkel set theory. Zermelo-Fraenkel set theory is one of several axiomatic systems that were proposed in the early twentieth century by the German mathematician Ernst Zermelo (1871-1953) and german born Israeli mathematician Abraham Fraenkel (1891-1965). Zermelo-Fraenkel set theory is an axiomatic attempt to formulate a theory of sets, which avoided certain set-theoretical aporia associated with the predicate of set membership, such as Russell's paradox.

The continuum hypothesis (CH) postulated by German mathematician Georg Cantor (1845-1918) – following observations made by Cohen and Gödel, CH is generally considered to be false – can be intuitively thought of as the transfinite continuum of a discrete iterative count which does not terminate at the highest countable number. Cantor discovered a number of paradoxes associated with this idea of transfinite cardinal numbers.

The continuum hypothesis can be formally stated in two forms

(i) No set whose cardinal measure lies strictly between that of the natural numbers $\mathbb{N}$ and the real numbers $\mathbb{R}$. [3]

(ii) There is no infinite set with a cardinal number κ between that of the "small" infinite set of integers ω_0 and that of the "large" infinite set of real numbers $|\mathbb{R}|$ (originally proposed by Cantor).

Cantor conjectured that $2^{\omega_\alpha} = \omega_{\alpha+1}$ but he had no way of identifying cardinals between one transfinite cardinal ω_α and its successor 2^{ω_α}. Thus the Continuum Hypothesis (CH) is the statement $2^{\omega_0} = \omega_1$, while its generalized form (GCH) is the statement $\forall\alpha\ (2^{\omega_\alpha} = \omega_{\alpha\alpha+1})$. Cantor also could not show that there exists a cardinal measure $|\kappa|$ of some natural number κ such that

(iii) $\omega_\alpha < |\kappa| < \omega_{\alpha+1}$

In the Boolean-valued treatment of Cohen forcing I introduce in Chapter One, the possibly false statement $\omega_\alpha < |\kappa| < \omega_{\alpha+1}$ is of key importance.

In Badiou's nomenclature, transfinite numbers are 'indiscernible.' A part (subset) of a situation is indiscernible if there exists no statement in the language of Zermelo-Fraenkel set theory that can consistently describe an indiscernible. According to Badiou's early writing, the first-order logical language of Zermelo-Fraenkel set theory is the authentic ontological language of any situation. Under certain conditions, called forcing conditions, the indiscernible (possibly non-existent) transfinite cardinal κ can be named with terms that are 'understood' in the forcing language of the situation. This happens to be the

[3] Cf. (Cantor 1952, 104-109) " … $\aleph_0$ is the least transfinite number cardinal number. If a is any transfinite cardinal number different from $\aleph_0$, then $\aleph_0 < \mathfrak{a}$ … to every transfinite cardinal number $\mathfrak{a}$ there is a next greater proceeding out of it … and also to every unlimited ascending well-ordered aggregate of transfinite cardinal numbers, $\{\mathfrak{a}\}$, there is a next greater proceeding out of that aggregate in a unitary way." Note: for all intensive purposes the least transfinite cardinal $\aleph_0$ has the same cardinal value as $\boldsymbol{\omega}_0$.

language of the complete models of Zermelo-Fraenkel set theory adjoined to certain *generic* extensions. Although I do not use discrete set-theoretical models, the idea that the names of indiscernibles can be meaningfully associated with indiscernibles forms a crucial conceptual foundation on which I will ground the notion of an abstract automaton subject.

Following the contributions of Kurt Gödel (Gödel and Brown 1940) and Paul Cohen (Cohen 1963) it can be shown that the continuum hypotheses can be neither disproved nor proved using the axioms of Zermelo–Fraenkel set theory. Cohen's method of forcing demonstrates the independence of the continuum hypothesis (Cantor 1952; Cantor and Jourdain 1915) from Zermelo-Fraenkel set theory. An ontologist (Badiou's mysterious entity) can construct a countable transitive model of set theory, which Badiou refers to this as a *quasi-complete situation* (Badiou 2005, 358-362)). By by generically extending this model, an ontologist can implement certain functional operations from which certain properties of transfinite orders can be deemed meaningful in quasi-complete situation. A subject, on the other hand, discerns only the contents of the quasi complete situation, without recourse to the functional mechanisms. A subject may nevertheless force the situation in which some truth-based property of transfinite indiscernibles can be deduced. This is enough to show that the impossibly large ontological character of an event can be made meaningful in the finitude that closes over an evental site.

Intuitively, forcing consists of generically expanding the set-theoretical universe, denoted V (c.f. (Gödel and Brown 1940, 47)) to a larger universe V^*. In this bigger universe, for example, one might have uncountably more parts than V. While this may appear impossible, it is nevertheless one of the Cantorial paradoxes associated with the continuum hypothesis. If we take $V^* = V \times \{0, 1\}$ and then identify some $z \in V$ with the partial order $\langle z, 0 \rangle$, an expanded membership relation involving a naming function can be introduced to fashion the list of ordered pairs of the form $\langle z, 1 \rangle \in V^*$. The pairs in this list can 'name' the objects included in the expanded universe even though these objects are indiscernible in V. Although very little is known about the indiscernible elements of V^*, the fact that a name can be associated with each indiscernible means that at least some 'hidden truth' associated with the object can be known. For example, the ordered pairing $\langle z, 1\rangle$ of object z and the singleton 1, says something about a 'truth' associated with z. By giving the indiscernible z a discernible name $\check{z}$ that is meaningful in V, then under certain forcing conditions, we can see that in actuality $\check{z} = z$. By analogy, even though an indiscernible is in itself meaningless, the fact that an it can be named is enough for the subject to be become, according to Badiou, the "finite instance of truth" who "saves the singular."

While the principles of forcing are usually applied to the universe of sets and the models of Zermelo-Fraenkel set theory, forcing principles will also hold for any situation that includes a self-reflexive singularity. Consider a situation S. Under the sign of the event, S can be generically extended and defined $S^* = S \times \{\mathbf{0}, \mathbf{1}\}$. Multiples which are members of the situation S, by definition, are also presented in S. Any multiple s presented in S, is discernible. Now, consider a Boolean algebra $\{\mathbf{0}, \mathbf{1}\}$ to have greatest and least elements **1** and **0** respectively.

A Boolean partial ordering of S can be thought of as a B-transcendental (or Boolean-transcendental) order of S. This procedure, however, should not be confused with Badiou's existential notion of the transcendental order of a world. Badiou's 'transcendental' is existential, while the B-transcendental order of S that I am describing, is purely ontological.

Consider an ultrafilter over a B-transcendental order of S to be a function that cannot be enlarged nor diminished and will alway return **1**. In this sense a Boolean ultrafilter is maximal. Any multiple $s \in S^*$ where S^* is a generic extension of S, is indiscernible if and only if $s \notin S$. We interpret a name for s to be the set $\check{s} = \{t : \langle t, p\rangle \in s \text{ and } p \in S^*\}$. Once a generic ultrafilter over B has been fixed – remember, an ultrafilter only returns the value **1**

– then S^* will be the set of names $\{\check{s} : s \in S\}$ such that $\check{s} = \langle s, 1 \rangle$. This means that any name that can be understood in a situation can also be used to name an indiscernible element in S^*, if the indiscernible it names is associated with the highest element of the B-transcendental of S.

Consider Badiou's puissant proposition:

> … the one is and that the pure multiple – inconsistency – is not. This is entirely natural because an indeterminate situation, not being the presentation of presentation, necessarily identifies being with what is presentable, thus with the possibility of the one … being is being in the possibility of the one. … being is being in the possibility of the one. (Badiou 2005, 52-53)

This powerful assertion expresses in natural prose, what the schemata of transfinite forcing can express with mathematical precision: the peculiarity of the event (which is pure ontological inconsistency) presents that which is unpresentable – the adjunction [4] of the one and the nothing – thereby allowing an indiscernible to be discerned. The event is the paradoxical condition in which the void of pure ontological inconsistency is, under the name of the one, presented as the saturated self-inclusive totality of every multiple in the situation. Of course, the one and its ontological dual the void, are referentially distinct under any normal sense-reference (or signifier-signified) distinction. 'Normal' situations are those in which the aporetic presentation of presentation does not occur. This leads to the thought that each of the signifiers 'void' and 'one,' signify two distinct signifieds. However, in the moment of the event and under the subjective name "nothing," the two distinct signifieds, are indeed, one. The evental adjunction of absolute ontological plenitude (pure being in itself) and the void (absolute existential nothingness), is completely inconsistent in consistent ontological situations. Into this mix we introduce the generic operation of naming. Under the generic conditions of forcing, a specific

[4] Adjoint functors are an import category theoretic devices we will discuss in later chapters. The functor relationship of adjunction is an isomorphism (bijective equality) between two categories and a family of bijections. Technically, adjointness is an equality relation.

naming function can be defined. In a generic naming operation, the plenitude of the one "which is," and the void "which is not," are synthesized in the absurdity of a singularity which simultaneously "is and is not."

Let **1** and **0** denote the greatest and least elements of the Boolean algebra $\{\mathbf{0}, \mathbf{1}\}$. **0** and **1** can also denote the least and greatest elements of the partial ordering of the situation *S*. Categorially, **0** is the initial object of the category of initial algebras, while **1** is a terminal object in the same category. The idea of the initial and terminal objects of the category of initial algebras and terminal coalgebras will be of importance in later chapters. The algebra-coalgebraic duality is fundamental to the idea of a lower-bound void which will be associated with the name 'nothing.'

Care should be taken not to conflate the algebraic *B*-transcendental of a situation, which is purely ontological, with Badiou's algebraic notion of the transcendental of a world, which has existential application. A *B*-transcendental is a classical Boolean partial order, whereas the notion of 'transcendental,' in *Logics of Worlds*, is a partial ordering of an intuitionistic Heyting algebra.

Let the "proper name of being" (ibid. 52-59) be the name $\varnothing^*$ and denote the proper void "$\varnothing$." Under the conditions of forcing we 'ontologically locate' – this is not a topological location in the sense of Grothendieck – the void $\varnothing$ at the highest point of the *B*-transcendental order of *S*. This location is the upper limit of the ordinal count of *S*. Mark this point **1** and call it the ontological site of the upper-bound void $\langle \varnothing, \mathbf{1} \rangle$. Conversely, the ontological location of the lower-bound void is expressed $\langle \varnothing, \mathbf{0} \rangle$.

Because Boolean ultrafilter is a function on a Boolean algebra that cannot be enlarged, it will always returns **1** (whereas a Boolean prime ideal always returns **0**). Consider **1** to be a generic ultrafilter [5] on B, then associate with each element $s \in S$ a name $\check{s} \in S^*$ such that $\check{s}^{S^*} = s$ for any generic extension S^* of S. Again, let $\mathbf{1}$ be the greatest element of B and let the name $\check{s}$ be the set $\{\langle \check{z}, \mathbf{1} \rangle : z \in S\}$. Substituting the name of the void $\varnothing^*$ for $\check{s}$ and $\varnothing$ for s, it turns out that under certain forcing conditions (an event) the name of the void $\varnothing^*$ is indeed $\varnothing$ in the generic situation S^*. This occurs when a name in the situation S is located at the edge of the void, which happens to be the highest point $\mathbf{1}$ of the B-transcendental order of S.

* * *

In Chapter One I explore an *unramified* treatment of forcing, which defines forcing conditions as partial orders of certain Boolean-valued models of Zermelo-Fraenkel set theory. This treatment, which is due to Kunen (Easwaran 2005b; Kunen 1980), is effectively equivalent to Cohen's earlier forcing techniques (Cohen 1963, 1964) now called ramified forcing. However, I will develop a situation-specific theory of generic forcing. These procedures introduce a number of structural properties that will be useful when considering the idea of an abstract coalgebraic body in later Chapters Two and Three.

The Upper and Lower Bound Edge of the Void

In *Being and Event*, Badiou draws a distinction between the virtual power of the state of representation of a situation and the actual ordinal count of the multiples presented. The (cardinal) power of the state is always in excess of the normative (ordinal) count of the situation. Badiou's 'meta-ontological' template for this notion is the power set axiom of

[5] In a mathematical sense, the "supernumerary letter" denoted "♀" in *Being and Event*, is conceptually equivalent to a generic ultrafilter for "the indiscernible ♀." (Badiou 2005, 356-357)

Zermelo-Fraenkel set theory. The cardinal power of any set X with n elements is 2^n, the cardinal number (or measure) of X.

Let the set $\{1, 2, 3\}$ be the elements of a set X of natural numbers. We can think of X as a situation. The power set of X, denoted $\mathcal{P}(X)$, is the set of the subsets of X. There are denumerably $2^n = 2^3 = 9$ distinct subsets in X.[6] Therefore, $\mathcal{P}(X) = \{\{1, 2, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1\}, \{2\}, \{3\}, \varnothing\}$. Clearly, there are more 'represented' parts (subsets) of X than there are 'presented' elements. Under Badiou's thinking, there are three elements *presented* in X, but there are nine parts *represented* in X. The cardinal number (or measure) of X – usually denoted $|X|$ – is nine, the number of its distinct parts. The character of transfinite cardinal measures are of considerable importance to Badiou's earlier thinking of the event and the subject. The notions of *errancy of excess* of a state of representation, Cohen forcing and inaccessible cardinal measures also dovetail into the principles of the power set axiom. The power of representation of a multiple is always in excess to that which is merely presented. For technical reasons (which are not entirely clear) any power set of a multiple must also include the empty set $\varnothing$ as a part. It follows that $\varnothing$, while not *presented* in the situation $X = \{1, 2, 3\}$, is nevertheless *represented* in its state $\mathcal{P}(X)$. Badiou is quick to point out: "… it can be shown that the necessity of the state results from the need to exclude any presentation of the void." (Badiou 2005, 522) In Badiou's treatment of the relationship between presentation and representation, however, it is not entirely clear what "the void," marked by the sign of the empty set, is or where it is ontologically and existentially sits in the recursive upward continuum of sets that initializes with the empty set $\varnothing$.

There are at least three possible responses. "The void" object could:

---

[6] Badiou follows the direct French to English translation of 'subset' by using 'part' to describe the subset of some set.

(i) be ontologically located where $\varnothing$, the sign of "the subtractive suture of being," is actually presented (Badiou 2005, 66-69).

(ii) be ontologically defined by transfinite recursion [7] but is not discernible and therefore denotes the general super-numerically greater than ω_0. That is, the void is subjectively equated with inaccessible cardinal measures. As we cannot prove in Zermelo-Fraenkel set theory that inaccessible cardinals exist, then "the void" if it is to be identified by some supernumerary measure, is equally inaccessible.

(iii) be existentially located in a place where the ontological saturation of a multiple takes a maximal coefficient of appearance. That is, where the proper inexistent $\varnothing_A = \{A\}$ of some multiple A, appears in a world with a maximal coefficient of intensity (Badiou 2009a). That is, the void object exists absolutely.

If we consider void to be absolute ontological emptiness of being, then any existential "appearance" of void will be minimal. In other words, void is nonexistent. If on the other hand void is considered to be the proper inexistent of a situation, it is ontologically either $\varnothing$ or the proper inexistent $\varnothing_A$ of a multiple A.

In Meditation Five of *Being and Event*, Badiou is adamant: "There are not 'several' voids, there is only one void ..." Wrapped in the ontological straight jacket of **Set**, this assertion is perfectly true. With a mixture of strict set-theoretical axiomatics, Neoplatonic reasoning and a sleight-of-hand narrative, the name of the void is the name of the count. When a situation becomes an evental site, that count is **1**.

---

[7] The set of natural numbers α^+ is the least cardinal $>\alpha$. κ is a successor cardinal iff $\alpha = \alpha^+$ for some α; α is a limit cardinal iff $\kappa > \omega$ and is not a successor cardinal. ω_α is then defined by transfinite recursion. (Kunen 1980, 18, 30)

* * *

The *Comprehension Scheme* (or Zermelo's third axiom) can be defined by the first-order sentence:

(i) $\exists y \forall x\, (x \in y \Rightarrow x \in z \wedge \phi)$

with y bound in ϕ. Because the variable y is in the range of an existential quantifier, it can be thought of as being bound by 'existence.' However, the formula ϕ may have any number of other unbound variables. ϕ is also unique by the axiom of extensionality. [8] We define this "existentially" bound y as either

(ii) $\{x : x \in z \wedge \phi\}$ or $\{x \in z : \phi\}$

or if not y is not existentially bound in ϕ, any self-referential definition of sets is not eliminated. Such a set may be

(iii) $\exists y \forall x\, (x \in y \Leftrightarrow x \in z \wedge x \notin y)$

which would be inconsistent if $z \neq \emptyset$.

By the Comprehension Schema we can form any z such that $\{x \in z : x \neq x\}$, which is void (empty). By the axiom of set existence (i.e. $\exists x\, \{x = x\}$), some set z exists so there is a set, which having no members, is void. By extensionality, such a set is unique. (Kunen 1980, 11). Set-theoretically speaking, there is only one void.

Badiou gives an ontological proof of the unique existence of the empty set, and thus the void, in an equivalent but more eloquent form than I have abbreviated above (Badiou 2005, 66-69, 189-190).

---

[8] The variable ϕ is considered to be free (or unbound) if it is not bound by either the existential quantifier that binds y, nor the universal quantifier that binds x. Thus ϕ may contain any number of elements or parts. The dyadic operators "$\Rightarrow$" , "$\wedge$" and "$\Leftrightarrow$" denote material implication, logical conjunction and material equivalence respectively.

However, the existential location of the proper name of being still remains unanswered. "… a proper name requires its referent to be distinct" (ibid. 68)). Such a referent may indeed be an ontological object, albeit a nonexistent one.

* * *

Take two sets, α and β, such that $\beta \in \alpha$ and consider proposition:

(iv) "If β is on the edge of the void according to α, this is because no element of β is itself an element of α: the multiple α presents β but it does not present in a separate manner any of the multiples that β presents." (Badiou 2005, 186)

Expressing this in a sentence of first-order logic

(v) $\exists\alpha\forall\beta\ (\beta \in \alpha \Leftrightarrow \exists z\ (z \in \beta \wedge\ \neg z \in \alpha))$

we see that (v) is a rewriting of $\exists y \forall x\ (\ x \in y \Leftrightarrow x \in z \wedge x \notin y)$, the Comprehension Scheme (i) but where $x \notin y$ is a substitution instance of ϕ with no unbound variables.

If, as Badiou argues, α *presents* β*,* then trivially by inclusion β is also presented. By the proposition (iv), the intersection of α of β does not *separately* present any element $z \in \beta$ and $z \in \alpha$, then by Comprehension, the intersection $\alpha \cap \beta = \varnothing$ is trivially empty.

For example, in the world of natural numbers, let a set $\beta = \{1\}$ and $\alpha = \{1, 2, 3\}$. Clearly β is an element of α. By the definition of intersection

(vi) $\alpha \cap \beta = \{z : z \in \alpha \wedge z \in \beta)$

in which case the intersection of α and β is not empty, i.e. $\alpha \cap \beta = \{1\}$. We find in this case that β does *not* stand on the edge of the void regarding α. We define the null class

(vii) $\varnothing = \{x : x \notin x\}$

and the universe class

(viii) $U = \{x : x \in x\}$.

The only way the proposition $\alpha \cap \beta = \varnothing$ is true is if $\exists y \forall x\ (\beta \in y \Leftrightarrow \beta \in z \wedge \beta \notin y)$.

Now define a set $A = \{\alpha : \alpha \in \alpha\} = \{A\}$. Clearly A is an ontological singularity. Now characterize 'standing on the edge of the void' as the ontological proposition $\{\{\alpha\}: \{\alpha\} \cap \alpha = \varnothing\}$. We have tautologically defined some 'standing at the edge of an "upper bound" void' that is a singularity. Denote this singularity "$\varnothing_A$." Just as we assigned the proper name "void," (remember, "a proper name requires its referent to be distinct") to the "lower bound" empty set $\varnothing = \{A : A \neq A\}$. Defining $\varnothing_A = \{A : A \in A\}$ then clearly $\varnothing \neq \varnothing_A$. Clearly the edge of the void at which a subject is to stand is either $\varnothing$ or $\varnothing_A$, not both.

A state of representation that is exactly the power set of the null set, $\mathcal{P}(\varnothing)$, is equivalent to the conjunction of the singularity $\{\varnothing\}$ and the power set of the proper the universe class. As I did above, we can define the universe class $U = \{x : x = x\}$ and let the proper inexistent of a multiple A be a variable substitution instance of U (i.e. we let $\varnothing_A = \{A : A \in A\} = \{A\}$), then the power set of the null set, whose proper name is "void," is equivalent to the conjunction of the singularity $\{A\}$ and the state of representation of the proper inexistent, $\mathcal{P}(\varnothing_A)$. Thus, $\mathcal{P}(\varnothing) = \{\varnothing\} \wedge \mathcal{P}(\varnothing_A)$. The void is a part of every state but not necessarily an element in every multiple.

This is an elaborate way of demonstrating that while the ontological reasoning for the uniqueness of the void by the axioms of comprehension and restriction (foundation) is tautologically sound, the idea of a multiple on the edge of the void and the idea of the ontological suture-to-being rests in a self-referential anomaly and the set-theoretical absurdity under which an event occurs. Recall that for multiplies $\beta \in \alpha, \beta$ is on-the-edge-of-the-void according to α if $\alpha \cap \beta = \varnothing$.[9] By the definition of intersection, $x \cap y = \{z : z \in$

[9] This is surprisingly similar to one form of the axiom of separation, otherwise known as the axiom of foundation. Every non-empty set A contains an element B such that $A \cap B = \varnothing$, which means that A and B have no elements in common.

$x \wedge z \in y\}$, if β is an element of α then $\alpha \cap \beta = \beta$. If $\alpha \cap \beta = \varnothing$ then β is already void, thereby rendering the idea of edge-of-the-void to be ontologically circular. However, aporetic circularity and tautological absurdity is exactly that which characterizes the event. It is on the back of a tautology that the ideas of the-edge-of-the-void and the subtractive-suturing-to-being may be recovered with a sleight-of-hand shift of focus from an ambiguous ontological object to a proper noun: a name.

Without considering *how* a multiple sits at the edge of the void or how the sign of the empty set has been sublated as a proper noun, the whole enterprise must end in a state of ontological stasis. To a certain extent this stalemate is resolved in the operation of a transcendental functor and the localizing of a suture-to-being we find in *Logics of Worlds*.

The point of such an elaborate unpacking of Badiou's idea of a subject on the edge of a void is that the issue may be either sidestepped or conceptually put away forever. This backing off from the ontological straightjacket of discrete set theory allows us to move on to the conceptual space of a silent singularity and a pseudo-dialectic turn, which is not overshadowed by the metaphorical and allegorical power of the event. The idea of a fragmented subject may then be reconstituted in the flow of the *sinthôme* and instantiated in the fictional character of Maria Wyeth, the tragic protagonist of Joan Didion's 1970 novel *Play It As It Lays*.

Nothing

The name 'nothing' is a powerful placeholder. However, 'nothing' does not have the same referential meaning as 'void.' To understand this difference, care must be taken not to conflate the specific efficacies of the algebraic theories I summon to describe the two concepts. The dense atomless Boolean algebras used to construct the models of set theory in my discussion of Cohen forcing in Chapter One, do not have the same structural efficacies as the existential Heyting algebras that Badiou uses to schematize his theory of appearance in *Logics of Worlds*. In my discussion of forcing, I algebraically describe specific processes and partial orders associated with certain Boolean-valued models of Zermelo-Fraenkel set theory. In this way, classical algebras can be used to describe the sets and ontological models used to in Cohen forcing. Whereas, Badiou's treatment of the generic in *Being and Event*, summons classic set theory. On the other hand, Badiou's algebraic schematics in *Logics of Worlds* describe the real-valued coefficients of appearance which can be indexed to the atomic base-sets of certain topological structures. As such, pure ontological objects can be subjectively identified as algebraic coefficients of appearance. In my discussion of the *sinthôme* and mutant automata in Chapter Three, I explore some theoretical applications of coalgebras. The terminal coalgebras I use in this regard are the mathematical duals of the initial algebras associated with Badiou's notion of the transcendental orders of worlds. However, a close reading of both *Logics of Worlds* and *Being and Event*, reveals that the notion of 'void' has an ontological efficacy linked to the non-aporetic representation of being, while 'nothingness' alludes to the zero existential intensity of non-appearance.

By Zermelo's second axiom, *Axiom der Elementarmengen*, the void set ontologically 'exists.' [10] Under Badiou's theory of appearance, the void cannot 'exist' (or appear) in a world except in the moment when the force of an event turns a normal situation into an evental site. In the moment of the event, the void 'appears' as a strong singularity bearing a maximal coefficient of appearance. Otherwise, the void is nonexistent under Badiou's algebraic theory of appearance. This highlights an ambiguity between ontological "existence" and existential appearance.

---

[10] "**Axiom II**. Es gibt eine (uneigentliche) Menge, die "*Nullmenge*" 0, welche gar keine Elemente enthält. Ist a irgend ein Ding des Bereiches, so existiert eine Menge $\{a\}$, wleche a und nur a als Elemente enhält; sind a, b irgend zwei Dinge des Bereiches, so existiert immer eine Menge $\{a, b\}$ welche sowhol a als b, aber kein von beiden verschiedenes Ding x als Elemente enthält. (Axiom der Elementarmengen.)" Translation in part: "**Axiom II**. There exists [is] a (fictitious) set, the "null set" 0, which contains no elements … " (my translation). (Zermelo 1908, 263)

However, in evental sites in which the absolute conspicuousness of the event is truncated as a weak singularity, theoretical grounds emerge for the conceptual viability of a non-evental, non-sublated pseudo-dialectic turn. The trace of the pseudo-dialectic turn I call *sinthôme*. Under the *sinthôme* 'nothing' is nominally fixed to in the order of the symptom. I instantiate this idea in the tragic character of Maria Wyeth.

Following the suicide of her friend BZ and the hysterical reaction of another friend Helene who blames Maria for the death, Maria recalls the moment:

> *One thing in my defense, not that it matters: I know something Carter never knew, or Helene, or maybe you. I know what "nothing" means, and keep on playing.* (Didion 1970)

Maria Wyeth: the quintessential essence of nothing

Maria Wyeth, the protagonist of Joan Didion's novel *Play It As It Lays*, lives in Los Angeles. She is a failed Hollywood actor, debilitated by the cracked up logic of Hollywood success. The shallow and facile world of Hollywood success in which Maria Wyeth finds herself, has psychologically thickened Maria's emotional constitution to the point of terminal density. In the time between her abortion and the death of her friend BZ, the dense emotional viscosity that has slowed her world – her only relief is the self-reassurance she derives from continually drive the Los Angeles freeways – began imperceptibly to lighten. As she miraculously extracts herself from the Hollywood dream, she finds herself gravitating towards a profound new discovery: the meaning of nothingness. To her *friend* Helene, Maria's this signals a withdrawal into a state of emotional catatonia. The nothingness of Maria's emotional inertia is simply a sign that she was not capable of accepting the truth of failure. By not intervening in BZ's successful suicide attempt, Helene erroneously believes that it was Maria's selfish vacuity that had "killed" him.

The sadistic imperatives of Hollywood culture have corrupted her friends and turned them against her. Not only do they no longer like Maria, they continue to deeply wound her. At the point of absolute inertia, in a mood of complete self-worthlessness, Maria turns away. Silent and imperceptible, hers is a quiet revolution conducted privately on the solipsistic level of the symptom. It is a catatonic revolt against a system and its players who sucked her blood, and then when it didn't suit them, forced her to cut and flush her unborn child. Maria's abortion marked the point of no re-turn.

In Didion's narrative we find no explicit revolutionary event. Maria's condition is not sublated to a higher level of meaning. Her revolution does not emerge as a Badiouian event. Yet there is something in the profound simplicity of Didion's narrative that reminds us of Badiou's aleatory treatment of truth, but without the dramatic existential prominence of an event. Maria takes her chances and plays the odds and plays them as they lay. They are *her* choices and they exceed the will of others. In her defiance, she must continually test for fresh waters to flush the amniotic stagnation that was never allowed to flow with the joy of natural childbirth.

Maria had subtracted meaning from her life in order for it to continue. She had a name for this subtraction: "nothingness." In a state of social automatism, any libidinal fortitude that had not been devastated by her abortion, nor corroded by her numerous meaningless affairs, is extracted from her emotional stasis and is pointed away from the hell that is the company of others.

Later I will describe Maria's condition (her symptom) as an "unknown state space." This will then be characterized as the mutable dynamism of her residual will and reformulated as an abstract mutant automaton. Her tenacious will to go on, will be schematically linked to dynamic function which ranges over unknown state spaces. A functor operation can then be additively applied whereby names begin to adhere to parts of the space and thereby Maria's imaginary *other* body may be formed when the name for 'nothing' finally sticks.

Sinthôme

Schematically, Maria's pathematic trajectory is the terminal coalgebraic trace I call *sinthôme*. "*Sinthôme*" is the ancient spelling of the French *le symptôm*, which is equivalent in therapeutic meaning to the English "symptom." Unlike Badiou's idea of the evental trace, the trace I call *sinthôme* is effective only in as much as it may be localized in an abstract coalgebraic topos upon which a nominal body – in name only – may be inscribed. This coalgebra can be schematized as a functor category, similar in effect to Badiou's transcendental functor in a "normal" positively ordered world. However, unlike the subjectivizable body which Badiou defines as an existential projection of an atomic topos, the topology of the *sinthôme* has no material base. As it is with Maria's imaginary body, the topology of the *sinthôme* is purely abstract. Maria's abstract imaginary body symbollically replaces her actual body Maria let be defiled by others. Describing this in Badiouian terms, Maria's imaginary body has a sum-zero transcendental coefficient of appearance. Her pathematic body simply does not exist in a material world.

During one of his most difficult and dense period of teaching, Lacan introduced the idea of *sinthôme* (Seminar of 1975-76 "*Le sinthôme*"). In earlier seminars, Lacan viewed the symptom as something which is inscribed in the analytical process, as signifiers whose meaning are cyphers to be interpreted by reference to an unconscious structure. This idea was later superseded by the notion of the *sinthôme*. In his Seminar "*L'angoisse*" (1962-63), Lacan shifts this psycholinguistic focus on the symptom to the idea that the symptom is a topological trace of subjectivizable jouissance, in which the subject can organize the unbalanced conditions of his or her neurosis into some cohesive modality. This modality, which resists analysis, Lacan called *le sinthome*. [11] In a previous seminar, Lacan had introduced the notion of the Borromean knot which forms a continuous topological string weaving the Real, Symbolic and Imaginary (RSI) orders

[11] The spelling ambiguity between the *sinthôme* and *le sinthome* will be explained later.

into a complex three-ringed knot. If the string is cut, then the trace is lost and the cohesion of the subject unravels.

In his 1975-76 seminar Lacan points out that by adding the *sinthôme* as a fourth ring to the Borromean knot stabilizing the cohesion of RSI and any threat of unraveling is allayed. In the same seminar, Lacan considered the writing of James Joyce to be an extended *sinthôme*. "Faced in his childhood by the radical non-function/absence of the Name-of-the-Father, Joyce managed to avoid psychosis by deploying his art as *suppléance*, as a supplementary cord in the subjective knot" (Evans 1996, 192).

Although Lacan's discussion of Joyce is an exegetic device, a "supplementary chord," in our reading of Didion's *Play It As It Lays*, the trace under which Maria Wyeth separates herself from others, enables her to emotionally stay intact while she inscribes meaning to her new imaginary "body." In the place where she abandoned her old body – the body which was symbolically overloaded and physically defiled by the will of the others, by the abortion ordered by her ex-husband Carter and in her disengaged sex with numerous partners – Maria is able to inscribe a turn. Indeed, her turning away can be schematized as an abstract coinductive functor operation of the *sinthôme*. From the edge of the void, she passes to the center of nothingness, to prevail beyond the imperatives of analytical meaning and her vulnerability to sadistic underbelly of Hollywood success:

> She had only the faintest ugly memory of what had brought BZ and Helene together, and to erase it from her mind she fixed her imagination on a needle dripping sodium pentathol into her arm and began counting backward from one hundred. When that failed she imagined herself driving, conceived audacious lane changes, strategic shifts of gear, the Hollywood to the San Bernardino and straight out past Barstow, past Baker, driving straight on into the hard white empty core of the world. (Didion 2005, 162)

The Going-On: Samuel Beckett's unnamable "I"

Against Maria's silent revolution, the protagonist "I" of Samuel Beckett's *The Unnamable* (Beckett 1958a) is bound by a more subtle and devastating logic than the cruel social logic of Maria Wyeth's Hollywood. Whereas the quiet coinductive force of the *sinthôme* drives Maria into smooth space at the edge of nothingness, Beckett's "I" is burdened by the legislative tyranny of overstated reason. Because there is on turning away, not is there sublation, there is no escape other than his ineffective struggle and the burden of going-on. Nor is there escape in silence. The emotional edge of social reason from which Maria turns, is for Beckett's "I," a forbidden aporetic zone beyond whose borders an ill-formed universe lays in silence. This forbidden no-go zone appears as the silence of a void he desperately seeks. Beyond this border "… it dies away in the vault, it calls that a vault, perhaps it's the abyss, those are the words, it speaks of a prison, for me alone, or waiting for me." (ibid. 172) "I"'s going-on becomes bound by the deadening grey/black analytical tyranny of reason. In the grey/black zone, an unidentifiable alterity he calls "them," forbids him to enunciate the cypher that would secure his escape. Badiou:

> The localization by the grey black ultimately entails that the being of being cannot be said as an isolated singularity, but only as void. When the fiction that fuses wandering and the darkness of immobility operates, we notice that what this place presents as the form of being can only be named 'the nothing,' or 'the void,' and has no other name. [12]

The inductive logic of the onward trajectory that forces his going-on, can only *terminate* at the edge of finitude. According to the aporetic logic of the event, when a subject forces the indiscernible stuff of transfinite excess to be presented as the generic evidence of truth, the proper name of being – "void" – will be understood. Beckett's "I" is caught in a state of paranoiac solipsism, because "they" forbid the one name to be spoken. There is no one else to speak it.

---

[12] "L'écrititure du générique: Samuel Beckett," in *Conditions,* Paris: Editions du Seuil 1992. Published also in English in (Badiou 2003, 7; 2008)

For Badiou, the proper name of being can never be "said" because there is no complete, consistent world in which being could be evident, except for the moment of the event. Beckett's "I" is wracked with paranoia, because *they* forbid the name being said and this leads him to the eternal pain of continuance and utter compliance to the reason of "their" will.

Beckett:

> I'm ready to be whatever they want, I'm tired of being matter, matter, pawed and pummeled endlessly in vain. Or give me up and leave me lying in a heap, in such a heap that none would ever be found again and try and fashion it. But they are not of the same mind, they are not of the same kidney and yet they don't know what they want to do with me, they don't know where I am or what I am like, … No, I didn't think anything and it didn't say anything to myself, I did what I could, a thing beyond my strength, and often for exhaustion I gave up doing it, and yet it went on being done, the voice being heard, the voice which could not be mine, since I had none left, and yet which could only be mine, since I could not go silent, and since I was alone, in a place where no voice could reach me." (Beckett 1958a, 84, 153)

In a Badiouian sense, the protagonist of *The Unnamable* is caught in a place where the state of representation that gives meaning to his world, is the errancy of excess of the power of its own presentation. The state has no power to present the *nothing*, whose subtractive being continues to haunt it. "… being as void 'inexists' for language, subtracted as it is from every *degree*." (Badiou 2003, 8) If "I" were to be the subject of an event, "I" could have named the unnamable and thereby changed the world around him. Indeed, Beckett's "I" stands close to the edge of the unnamable void, but "I" has neither the body nor the will to cross over. The juridical language of consistent reason cannot provide the analytical tools for his reprieve. Generic truth cannot be named without entering the generic no-go zone of the indiscernible, nor crossing the aporetic edge of logical consistency.

In the world of *The Unnamable* there is no ‘upper-bound’ void at whose edge a subject may stand. Nor is a void to be found in the inductive logic of well-founded reason. Against a background of cold war anxiety, Beckett constructs a narrative of an ‘out there’ alterity that would give evidence of the terminal collapse of a world turned back upon itself. If only the truth could be said, let alone heard.

Without the turn nor the event, there is only continuance in the logic of the going on.

In *The Unnamable* world of Beckett’s “I” there is no *sinthôme* under which to gather a new subjectivizable body upon whose surface meaning may be inscribed. This is not the case for Maria Wyeth. While the “actual” world of her social milieux is one of horror and resentment, Maria enters the cool zone of the *sinthôme*. Helene despises Maria for the death of BZ and Carter is continually frustrated at Maria’s inability to take care of herself. Yet something turned in Maria’s world, something significant. A new abstract body emerges. Following the trace of the *sinthôme*, this new body reconfigures Maria's situation and constructs itself in a form that is *bisimilar* to the one that Maria had emotionally abandoned.

Ill-founded Worlds

In set theory, non-well-founded sets can be described as aporetic instances of self-inclusion entailed by certain set-theoretical aporia. On the other hand, non-well-founded sets may be included in coalgebras that are mandated by an anti-foundation axiom. British mathematician and computer scientist Peter Aczel is known in the mathematics community for his work in non-well-founded set theory. In his discussion of non-well-founded sets in (Aczel 1988, 6; Fernando 1994; Kunen 1980, 6), Aczel introduces a number of anti-foundation axioms, due to Maurice Boffa (Boffa 1969, 16-56), Paul Finsler (Finsler 1975) and Dana Scott (unpublished paper from the Stanford Congress of Logic, 1960). (Aczel 1988, 59, 48, 49) (Fernando 1994).

The inclusion of non-well-founded sets in consistent set theory is foreclosed by the axiom of foundation. This idea is historically due to the Russian-born logician Dimitry Mirimanoff (1861-1945). In Mirimanoff's view a set, x_0, is well-founded if and only if it has no infinite descending membership sequence. Mirimanoff:

> Let *E* be a set, *E** one of its elements, *E*** any element of *E**, and so on. I call a *descent* the sequence of steps from *E* to *E**, *E** to *E***, etc. ... I say that a set is *ordinary* when it only gives rise to finite descents; I say that it is *extraordinary* when among its descents there are *some* which are infinite. [Quoted in (Aczel 1988, vii)][13]

Given the descending well-founded set sequence

$$\ldots \in x_2 \in x_1 \in x_0$$

there will always be some element of the sequence, which terminates the descent. On the other hand, a set is non-well-founded if and only if it is extraordinary in the sense of Mirimanoff.

A well-founded situation will always ontologically foreclose any anti-foundation axiom, except when the situation is the site of an evental. In the sense of Mirimanoff, sites are extraordinary sets, while normal non-evental situations are not extraordinary.

The theory of non-well-founded sets has been applied in the logical modeling of non-terminating computational processes in computer science and final semantics. Coalgebras are often construed as final or terminal coalgebras where coinductive destructor/observer operations are important, while dually, classical algebras are can be initial algebras, in which inductive constructor operations are important. Constructors generate data form initial principles, void, empty list etc. to which a prefix or successor operation generates meaning. On the other hand, destructor, or observer, or transition functions can be applied

[13] Mirimanoff, Dimitry (Дмитрий Семёнович Мириманов)(1861–1945): *Les antinomies de Russell et de Burali-Forti et le ploblème fondamental de la theorie des ensemble* (1917). Quoted in (Aczel, 1988, vii)

to the terminal infinite objects generated by constructors. Conversely, infinite terminal objects are the starting point of coinductive coalgebraic operations. These operations typically involve the dynamic observer functions, I call, **`go-on`** and **`name`**. The function **`name`** makes a direct observation of a possibly infinite unknown state space and returns the name 'nothing' if the name sticks, otherwise the command **`go-on`** activates and the automaton moves on.

I schematically define an abstract mutant automaton as a dynamic (or mutable) functioning instance of a coalgebra, (Adámek 2005) which operates under the coalgebraic logic of a cotranscendental order. A cotranscendental inverts the transcendental logic of any Badiouian world.

In the case of Maria Wyeth, the occurrence of *silent singularity* and a *pseudo-dialectic turn* legitimizes the transgressive impetus of her imaginary world. We should note however, that the maximal evental appearance of the what Badiou refers to as a strong singularity, is an 'onto-logical' (Badiou 2006)) instance of a non-well-founded multiple. Alternatively, strong singularities can be thought of as the maximal appearance of proper inexistent of a multiple (Badiou 2009a, 341-343). The place of the terminal object of an initial algebra is exactly the site of an event. The terminal object of an algebra is the site of the turn. Singularities form sites, but only strong singularities bear the maximal appearance of an event. The singular entities Badiou calls weak singularities, I call silent singularities. Silent singularities are existentially nonexistent in a transcendental sense of badiou, but they nevertheless form sites. The *sinthôme* flows from the *nonexistent* occurrence of a silent singularity.

Precipitated by the turn, Maria's will to go on, subsumes her world. This world is *bisimilar* to the one Maria had emotionally abandoned. I schematize Maria's will to go on as a coalgebraic mutant automaton in an unknown state space. [14]

The Pseudo-Dialectic Turn

The formation of a post-evental body that retroactively bears the signs of real change, does not hold in a cotranscendental order. Although the occurrence of a silent singularity forms of the site of a pseudo-dialectic turn, this site is not evental. Events, according to badiou, entail some sort of materialist dialectic sublation. Because there is no synthesis of being and appearance at the site of a pseudo-dialectic turn, the flow of "truth" entailed by the *sinthôme* must be ratified by the principle of mutable observation. This "truth" is not retroactively constructible. Unlike the *constructor* algebras of the partially ordered worlds we find in *Logics of Worlds*, terminal coalgebras are obtained from pure observation (c.f. (Jacobs and Rutten 1997, 3)). The basic dichotomy of algebraic and coalgebraic orders can be described as a 'constructor' principle versus a 'destructor' principle. Under the *sinthôme*, the coalgebraic radicalization of a world involving a silent singularity, will invert the transcendental logic of any onto-logical situation. A world that has turned is ontologically ill-founded and existentially transgressive.

The pseudo-dialectic turn involves no sublation (*Aufhebung*), nor any material switch to a new dialectical plane of reason. Nor does the turn entail some Cohen-forced generic expedition into the indiscernible transfinite domains of the constructible universe (Badiou 1988). As it is with any Badiouian event, a subjectivizable body is formed in the trace generated by the evental occurrence of a strong singularity. On the other hand, the idea of the pseudo-dialectic turn involves a radical inversion of a legitimate transcendental

[14] The mathematical template for the sort of automata I am proposing are, formally speaking, abstract mathematical machines designed to operate with infinite data states. (Jacobs 1997; Park 1981, 167-183)

ordering of a world into the transgressive cotranscendental trace that is the *sinthôme*. The relationship between the actual world of the transcendental order and the parallel world of the *sinthôme* is a bisimulation. Bisimulations are intensional or affective equivalences, whereas the ontological equivalence of two situations is extensional and effective.

Bisimilarity

In mathematical logic, particularly Kripke semantics, modern computer science and the semantics of programs, the binary relation *bisimilarity* (which I denote "$\approx$") establishes the behavioral equivalence of transition states. Partial orders, such as the transcendental order of a world, may be defined transitionally. In *Logics of Worlds*, Badiou defines a relation between two intensities of appearance in a world, as degrees of difference, or points, on the transcendental of that world. These points are ordered by the 'greater than or equal to' relation $\leq$. Any multiple existentially indexed by a transcendental degree has either 'a greater than or equal to' intensity relation with any other multiple. Under the transcendental order $\leq$, multiples *exist* either differentially or equally. In a cotranscendental order, this relation is defined as the bisimilarity of transitional states under the flow of the *sinthôme*. Two transitional states are bisimilar if a transition between them preserves the behavioral affects of one state in the other. In this sense, preserving a property means that any transition between states, the properties of the source state remain invariant in the transition to a target state. I will define the cotranscendental function going-on as a dynamic *named transition system* under the mutable flow of the *sinthôme*.

In naïve set theory, the axiom of extensionality embodies the idea that a multiple (set) is composed entirely of its elements. In turn, elements are determinable if and only if each element shares the same predicate value. By the axiom of extensionality, two or more sets are equal when they share the same members.[15] On the other hand, the bisimilarity of two multiples is established in quite a different way.

By the axiom of extensionality, every multiple in a situation is well-founded. However, when a situation becomes a site with the occurrence of an ontological exception, the subtractive other (or the proper inexistent of the situation) is also included in the situation as an element. The subtractive alterity of a situation, is non-well-founded. In set theories, which admit non-well-founded sets, the notion of equality between sets is radically reconsidered. Extensionality guarantees that equal multiples are ontologically indistinguishable in any situation. However, care should be taken when considering that two multiples, which are ontologically equivalent, may 'appear' in a world with different existential intensities.

Under the transgressive sign of the turn, the legislative tyranny of the axioms classical set theory and the legitimate notion of equality, is challenged when non-well-founded multiples are admitted. An event is the momentary occurrence of a non-well-founded multiple. As we have seen, non-well-founded sets have infinite depth. Intuitively, non-well-founded sets are allowed to contain themselves and thereby violate the axiom of foundation. This axiom forbids the formation of multiples which entail infinite descending sequences

$$\dots\ x_n \in x_{n-1} \dots \in x_2 \in x_1 \in x_0$$

A set S which satisfies the equation $S = \{S\}$ (i.e. a set which is equal to the set which includes itself as its only member) is logically circular and ontologically non-well-founded. A situation S is a site if it admits the singularity $S = \{S\}$ (if only momentarily)

[15] In symbols, Zermelo's first axiom: the axiom of extensionality can be expressed by the first-order sentence $\forall x \forall y (\forall z (z \in y \Leftrightarrow z \in x)) \Rightarrow x = y$. When discussing the various axioms of set theory, I will use a notation of first order predicate logic in which "$\in$" denotes the predicate of set membership, "$\Leftrightarrow$" denotes material equivalence, "$=$" denotes simple equivalence and "$\Rightarrow$" denotes material implication. My formal description of the Zermelo-Fraenkel axioms follows (Kunen 1980; Lemmon 1968)

and is therefore non-well-founded. under the axiom of extensionality Take two situations S and S' and restrict the definition of each to two sets which contain the other as their only member. This means that $S = \{S'\}$ and $S' = \{S\}$. Any attempt to establish that the two equations are consistently equal under set-theoretical extensionality will end in the ineffective tautology "S and S' are equal if and only if S and S' are equal."

Intuitively, the axiom of foundation allows for the ontological construction of multiple situations by transfinite induction. Badiou's ontology of multiples, situations and events is inductively reasoned. Even Badiou's notion of the post-evental construction of a subjectivizable body involves retroactive induction on the topology of a post-evental situation.

If the pseudo-dialectic turn had the same ontological properties as the event, then the idea that the transgressive imaginary world of the *sinthôme* is ontologically and transcendentally equivalent to the external world of the evental trace, is problematic. If the *sinthôme* is defined as a transgressive flow, then it is plausible to presume that the silent force of the turn inverts the inductive transcendental logic of the actual world into the coinductive logic of the *sinthôme*. My reasoning for this is expressed in the observation:

> *The axiom of foundation forces the universe of sets in which the other axioms (the basic axioms) should be interpreted to be the smallest possible one; i.e. to be an* [outer] *"inductive universe." By contrast, axioms of anti-foundation lead to the largest possible universe, i.e. a* [inner] *"coinductive universe." Indeed, referring to the algebraic/coalgebraic interpretation of induction/coinduction, the foundation axiom can be seen as expressed as a requirement that the universe of sets should be an* initial algebra *for the powerset* [sic.] *functor, whereas anti-foundation … can be expressed as a requirement that the universe should be a* final coalgebra *for the same functor. The former is an inductive definition of the universe, whereas the latter is a coinductive one. (Sangiorgi 2009a, 134)*

Terminal Objects

In *Logics of Worlds* Badiou describes the site of an event as a situation in which a singularity appears with maximal existential intensity. An evental singularity is always the ontological condition whereby the situation counts as an element of itself. In this sense, the appearance of a singularity is always an aporetic. "A site testifies to an intrusion of being as such into appearing." A "strong singularity" appears (or exists) to the maximal degree of intensity, while a "weak singularity" exists to a minimal degree of intensity (Badiou 2009b, 374-379). It follows, that a weak singularity does not "exist" in a world because its intensity of appearing is minimal (ibid. 374-379). Nevertheless, Badiou argues, a weak singularity constitutes a site (ibid. 395).

Schematically, the transcendental of a world is an algebraic partial order which closes under maximal and minimal degrees of intensity. Badiou denotes these coefficients "M" and "μ" respectively. Any two elements x, y of a situation in a world **m** under the transcendental T of that world, may be assigned a transcendental degree derived from a partial order relation over T. Thus, an identity coefficient for x, y is a point $p \in T$. When the transcendental identity function is defined $\mathbf{Id}(x, y) = p$ (Badiou 2009b, 243-245) we say x and y are identical to the degree p. The elements x and y are absolutely identical when $p = M$, and absolutely non-identical when $p = \mu$.

At this point of the discussion I will recast Badiou's "onto-logical" theory of the transcendental order of a world in category theoretic terms. This will help to formally distinguish the conceptual differences between Badiou's algebraic notion of a transcendental order of a world and my coalgebraic notion of the cotranscendental order of the *sinthôme*.

If the sets x and y are thought of as objects a and b of the category **Exist** and redefine Badiou's existential identity function $\mathbf{Id}(x, y)$ as the **Exist**-category product object $a \times b$

with projections $a \leftarrow a \times b \rightarrow b$, then we shall say $a \times b$ is the *initial object* of the transcendental T of a world **m**. Notice that Badiou's algebraic identity function is now expressed as a category object. Conversely, the 'co-product' $a + b$ of the objects a and b in the category **Exist**, together with the co-projections $a \rightarrow a + b \leftarrow b$ is the *terminal object* of T. Now, consider the transcendental T to have a terminal object $a + b$ and an initial object $a \times b$. As we shall see, the initial and terminal objects of a transcendental correlate to the minimal and maximal intensities of appearance under Badiou's notion of a transcendental order. These also correlate to the greatest and least fixed points of the *sinthôme*. Structurally speaking, the *sinthôme* is a functor category of coalgebras.

A categorial interpretation of Badiou's identity function involves the initial and terminal objects of the functor category of algebras and coalgebras, which in turn, will determine the initial and terminal objects of the *sinthôme*. As we hall see, the terminal object of the *sinthôme* is located at the least mutable point that can be fixed by a name. In the case of Maria Wyeth, the name 'nothing' *fixes* the mutable trace of the *sinthôme* at the least fixed point of the cotranscendental ordering of her imaginary world.

Given a categorial treatment of Badiou's notions of object and transcendental, we can formally describe the existence an object as the direct (Cartesian) product of an ontological multiple A and its transcendental index of appearing. The maximum intensity of the evental appearance of a singularity (its being-there) momentarily fixes the terminal object of T at its greatest fixed point. This 'unique' impossible object $A \in A$ (or $A = \{A\}$) characterizes its own "generic form of appearing" as the Badiouian object $\mathbf{E}(A, \mathbf{Id}) = M$. That is, the 'generic' existence of the object A, regarding the transcendental T, is the maximal existential value of appearance that can be indexed to the 'impossible' ontological object $A = \{A\}$. This is expressed in the equation $\mathbf{E}(A, \mathbf{Id}) = M$. This equation also expresses the idea that the singularity $A = \{A\}$ appears, if only momentarily, with maximal existential intensity.

If the terminal object of an algebraic order is both ontologically and existentially expressible, then the same object is simultaneously both the terminal object of a transcendental order and the initial object of the cotranscendental functor category I call *sinthôme*. This is because the functor category of final (or terminal) coalgebras reverses the arrows of the functor category of initial algebras, and vice versa. The categorial duality between coalgebras and algebras is essentially one of reversal. The initial objects of an initial algebra are the terminal objects of a terminal coalgebra. If the *B*-transcendental trajectory of a situation (i.e. the ordinal count of a situation) terminates at the point I call the pseudo-dialectic turn, then its coalgebraic dual trajectory, the *sinthôme*, begins at the same point.

Whereas Badiou's subject-body (Badiou 2009b, 455-491) exists transcendentally, Maria's *new* mutant automatonic body, the one that *replaces* her emotionally spent over-socialized actual body and *coexists* with her actual body, does so on the level of the symptom. Maria sidesteps her emotional adversaries – her ex-husband Carter, Helene and the others – and the trumped-up imperatives of Hollywood success. By naming 'nothing' in the tumultuous emotional *Fluß* where a name for 'something' could not adhered, Maria marks the terminal object of the cotranscendental order of the *sinthôme*.

* * *

We can categorially expand the transcendental - cotranscendental dichotomy by specifying a categorial version of Badiou's notion of an evental transcendental functor. We can define a *transcendental functor* category:

$$\boldsymbol{F}\uparrow: \mathbf{Exist} \rightarrow \mathbf{Set} \times \mathbf{Exist}$$

Dually, the schema of the pseudo-dialectic turn is the *cotranscendental functor* category:

$$\boldsymbol{F}\downarrow: \mathbf{Set} + \mathbf{Exist} \rightarrow \mathbf{Coexist}$$ [16]

[16] **Coexist** is the coalgebraic dual category of the category **Exist**.

Note: the arrows of the transcendental functor category $\boldsymbol{F}\uparrow$ are reversed in the coalgebraic cotranscendental functor category $\boldsymbol{F}\downarrow$. Remember, functors are function-like operations that range over categories, category objects and morphisms (arrows) and are therefore both more powerful and sophisticated than simple set-theoretical functions over multiples. The idea of a transcendental functor links the two categories, **Set** and **Exist**, in such a way as the product [17] and co-product objects of the functor categories can be categorially formalized as the product object $\mathbf{Set} \times \mathbf{Exist}$. This object is the terminal for the functor $\boldsymbol{F}\uparrow$. For example, Badiou's idea of an object, expressed as the ordered pair $\langle A, \mathbf{Id} \rangle$,[18] encapsulates the notion that any atomic or ontological object A can be indexed by the existential identity function **Id**. This means that A is an object in the ontological category **Set** and **Id** is an arrow in the category **Exist**. Consider Badiou's equation $\mathbf{E}\langle A, \mathbf{Id} \rangle = M$: we can 'categorize' this by letting **E** denote the existential function that assigns a transcendental degree in the category **Exist** to an object A in **Set**. When this is the case, the the pair $\langle A, \mathbf{Id} \rangle \in T$, exists under some degree of intensity $p \in T$ such that $T \subseteq \mathbf{E}$.

Let the glyph 'nothing' take the schematic form $\mathbf{1} + (A \times X)$. Let the singleton object $\mathbf{1} = \{X\}$ be an initial object of a coalgebra. Let "A" denote an observable set of (naming) behaviors, and let "X" denote an unknown state space. Let "A^*" denote a set of 'as yet unnamed' behaviors in A and let A^∞ be the infinite list of possible observable behaviors.

---

[17] In the category **Set** the direct (Cartesian) product $a \times b$ is only defined up to isomorphism. This means that $a \times b$ will not hold ontologically in generic situations, but will hold as the greatest lower bound of a transcendental. Therefore, we will always speak of 'a' product of a and b and not 'the' product. See (Goldblatt 1979b, 47). The direct product of sets A and B is the list of all ordered pairs whose first component is an element of A and whose second component is an element of B. The direct product of two sets can also be used to define a binary relationship, such as Badiou's **Id** function. For example, for all $a \in A$ and $b \in B$, the ordered pair $\langle a, b \rangle \in \mathbf{Id}$ is notationally equivalent to $\mathbf{Id}(a, b)$ where $\mathbf{Id} \subset A \times B$.

[18] I use the angle brackets to denote the ordered pair $\langle A, \mathbf{Id} \rangle$, whereas Badiou uses the convention of rounded brackets to denote the same order. $(A, \mathbf{Id})$ Both conventions appear in the literature.

An abstract mutant automaton can be structured as a coalgebra of the functor $T(X) = \mathbf{1} + (A \times X)$ with a base carrier set $A^{\infty} = A^{*} + A^{\infty}$ of finite and infinite lists of A's. Thus the going-on of the cotranscendental order of the *sinthôme*, can now be schematically described as a "possible-next" function $\mathbf{pn} : A^{\infty} \to \mathbf{1} + A \times A^{\infty}$ of a terminal coalgebra (c.f. (Jacobs 1997, 26)).

In a world existentially ordered by an algebraic transcendental, Badiou's definition of the envelope of two multiples, is the structural equivalent of the *least* upper bound (l.u.b.) of two sets p, q, which we denote "$p \sqcup q$" (Badiou 2009a, 163-165). We shall see that the least upper bound of two set objects $p \sqcup q$ may be categorially defined as their co-product object, $p + q$.

If we formally define the transcendental of a world as the partial order $\langle \mathbb{P}, \leq \rangle$, then $p + q$ can be algebraically defined as the analog of $p \sqcup q$:

(i) if $p \leq p + q$ and $q \leq p + q$ then $p + q$ the upper bound of p and q

(ii) if $p \leq c$ and $q \leq c$ then $p + q \leq c$, i.e. $p + q$ is the less than any other upper bound of p and q (Goldblatt 1979a, 55, 179).

In other words, the co-product $p + q$ of two category objects in **Set** is an analog of Badiou's definition of the envelope of the parts of a multiple. This detailing will be crucial to our discussion of the distinction between the initial and terminal objects of the coalgebraic trace *sinthôme*.

Projection and Coprojection

The idea of transcendental *projection* is crucial to Badiou's functor-based conjoining of existence (appearing) and his materialist ontology. (Badiou 2009a, 166) However, we heed to further clarify what exactly the object $(A, \mathbf{Id})$ is category theoretic terms. As we have seen, the ordered pair (A, **Id**) the Cartesian product category **Set** $\times$ **Exist**. lays down formal grounds for the onto-logical synthesis of material being and existential appearance. If, as Badiou observes, "[o]ne 'reascends' from appearing to being showing how the atomic composition of an object affects the multiple-being underlying the object. … This means contrary to Kant's conclusions, that appearing authorizes real synthesis."

In set theory the direct product set of two sets A and B is defined

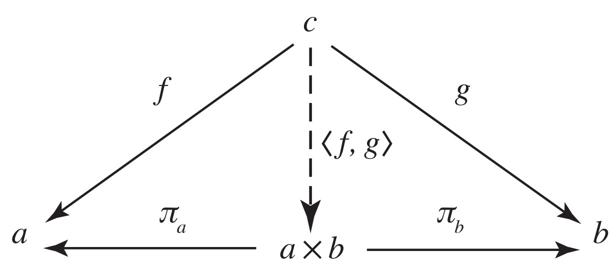
(iii) $A \times B = \{\langle x, y \rangle : x \in A \text{ and } y \in B\}$

This definition states that the (possibly infinite) list of all ordered pairs $\langle x, y \rangle$ for every $x \in A$ and $y \in B$, forms the product set $A \times B$. Each ordered pair can be thought of as share a certain relationship which is a subset of the product object. Intuitively, the ordered pair relationship may be characterized as x *before* y, or *whenever* x *then* y, and so on. If we denote the ordered pair relationship between x and y as R, then it is not difficult to see that $\langle x, y \rangle \in R$ and that R is properly included in the list that constitutes the product set $A \times B$, i.e. $R \subseteq A \times B$.

At the beginning of Book III in *Logics of Worlds*, Badiou introduces a 'function of appearing' as an indexes ontological sets to the existential values of appearing. The identity function **Id** (a, b) means to which every ordered pair $\langle a, b \rangle$ of elements $a, b \in A$, there corresponds an transcendental (existential) index of appearing, which is a value of the function **Id**. Therefore the function **Id**(x, y) can be unambiguously rewritten as the ordered pair $\langle x, y \rangle \in$ **Id**. In a general categorial sense, the functional relationship **Id** – which signifies the 'transcendental relationship' between set-theoretical (ontological) elements and algebraic (existential) indices – is a functor category **Id**$\uparrow$: **Alg** $\rightarrow$ **Set** $\times$ **Alg**

Product and Co-Product Objects

As we have seen, a Cartesian product is based on the set-theoretical definition of the product set by the first-order sentence $A \times B = \{\langle x, y \rangle : x \in A \wedge y \in B\}$. A product in a category of two objects a and b is an object $\langle a, b \rangle$ together with a pair of projections π_a : $a \times b \rightarrow a$ and $\pi_b : a \times b \rightarrow a$. By the rules $\pi_a\,(a, b) = a$ and $\pi_b\,(a, b) = b$ and for functions $f : c \rightarrow a$ and $g : c \rightarrow a$, there is a unique arrow $\langle f, g \rangle : c \rightarrow a \times b$ with the composites $\pi_a \circ \langle f, g \rangle = f$ and $\pi_b \circ \langle f, g \rangle = g$ such that the diagram



commutes. This categorially defines Badiou's identity function $\mathbf{Id}(a, b)$ as the unique arrow $\langle f, g \rangle : c \rightarrow a \times b$.

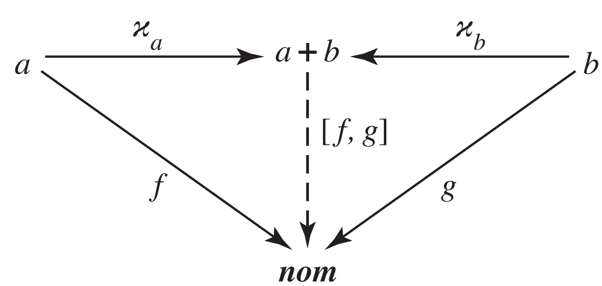
The product operation does not only apply to sets, but also to functions. For set functions $f : A \rightarrow A'$ and $f : B \rightarrow B'$ a function $A \times A' \rightarrow B \times B'$ can be defined by arrow $(a, b) \mapsto (f(a), f(b))$. This function can be written as $f \times g : A \times B \rightarrow A' \times B'$, upon which the symbol $\times$ becomes saturated, but it does show that the product $\times$ is also *functorial* (Goldblatt 1979a, 46-50; Jacobs and Rutten 1997, 11).

* * *

The categorial dual of the direct product object $A \times B$ is the disjoint union or sum $A + B$ object in **Set**. (ibid. 54; ibid. 11). Specifically:

(iv) $A + B = \{\langle 0, a \rangle : a \in A\} \cup \{\langle 1, b \rangle : b \in B\}$

The components 0 and 1 ensures the union is disjoint (having no elements in common). Instead of the projections π_a and π_b of (iii), we now have the coprojections (or injections) $\varkappa_a : a \to a + b$ and $\varkappa_b : b \to a + b$. Under disjoint union, we define the unique arrow **Id** (which we defined as the unique arrow $\langle f, g\rangle : c \to a \times b$) we define the co-product object as the unique arrow $[f, g]$. The arrow $[f, g]$ thereby inverts the unique arrow $\langle f, g\rangle$ we have associated with the existential identity function **Id**. The composition of the co-projections $\varkappa_a$ and $\varkappa_b$, means that trajectory of **Id** is reversed by moving from the disjoint union of disjoint objects (the maximal terminal object of **Id**) to a name-object ***nom***. Using the rules $\varkappa(a) = \langle 0, a\rangle$ and $\varkappa(b) = \langle 1, b\rangle$ and for functions $f : a \to$ ***nom*** and $g : b \to$ ***nom***, we see that there is a unique arrow $[f, g] : a + b \to$ ***nom*** and the composites $[f, g] \circ \varkappa_a = f$ and $[f, g] \circ \varkappa_b = g$ are such that the diagram

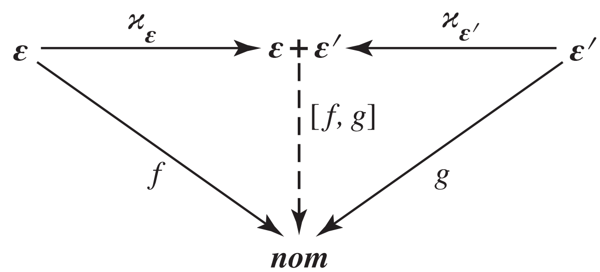


commutes.

In this way the unique identity arrow $\langle f, g\rangle : c \to a \times b$ of (iii) (which, as we determined, is equivalent to Badiou's identity function **Id**(a, b)), reverses to becomes the unique arrow $[f, g] : a + b \to$ ***nom***.

We will see that ***nom*** denotes a name-object assigned by a mutant automaton.

The co-product $p + q$ of two category objects in **Set** is an analog of Badiou's definition of the envelope of the transcendental of a world. By definition, the proper inexistent of a multiple is a non-well-founded multiple, whose maximal transcendental index marks the greatest fixed point of an existential order. The occurrence of a non-well-founded set can also be seen as the disjoint union of all other multiples: a singularity. In the instance of the terminal object being the disjoint union of two non-well-founded multiples $\boldsymbol{\varepsilon}$ and in $\boldsymbol{\varepsilon}'$ in the category **Set** and together with two coprojections $\varkappa_{\varepsilon} : \boldsymbol{\varepsilon} \to \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}'$ and $\varkappa_{\varepsilon'} : \boldsymbol{\varepsilon}' \to \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}'$, we see that the diagram



commutes under the same conditions as above.

In a general sense, the notion of ‘functor’ can be a mapping from one category to another, and not simply between sets. However, in our mathematical description of a mutant automaton we only need to consider functors which go from the category of sets and functions to themselves.

If we write $\boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}'$ as $\mathbf{1}$ but consider $\mathbf{1} = \{*\}$ to be the singleton set, with typical element $*$, then there is exactly one function $X \to \mathbf{1}$. We then say that $\mathbf{1}$ is *final* (or terminal) in the category **Set**. If the functions in $\mathbf{1} \to X$ corresponds to all the elements of X, and we write $\mathbf{0}$ for ‘the nothing,’ which we presume is a name for the empty set $\varnothing$, then for every set X there is exactly one function $\mathbf{0} \to X$. This property we will call the *initiality* of $\mathbf{0}$. On the other hand, the *finality* of $\mathbf{1}$ is a final (or terminal) object of an inductive algebra. The dual is the initial object of a final coalgebra. The sets $\mathbf{0} = \mathbf{0} \times \mathbf{0}$ and $\mathbf{1} = \mathbf{1} + \mathbf{1}$ can also be seen as the empty product and co-product objects respectively.

* * *

Although a silent singularity constitutes a site (our notion of silent singularity is structurally equivalent to what Badiou refers to as 'weak singularity' (Badiou 2009b, 389-396)), it nevertheless marks the non-appearing of being as the pseudo-being-there of the symptom. The affects of the pseudo-dialectic turn are then traced along the trajectory of the *sinthôme* by the mutant automaton we call subject. As an effect of this pseudo-being-there, the empty plenitude of the silent singularity marks the pathematic beginning the *sinthôme*, the trace of which terminates in the nominal object we call 'nothing':

> "Tell me what matters," BZ said.
> "Nothing," Maria said. (Didion 2005, 202)

Soon after, BZ lays dead, having killed himself by overdose.

Maria said "nothing" and BZ *said* nothing. In so doing, BZ ignored the nominal power of the *sinthôme* and confused it with the "absolute non-self-identity" of "the axiomatic of death" (Badiou 2009a, 269) and turned away from the life that Maria held intact under the flow of the *sinthôme*.

Chapter ONE

Voids

The Void is (Not) Necessarily Unique

Badiou's ontology is bound by legislative power of the axioms of classical set theory. By the axiom of the empty set, the void exists. By extensionality, the void is unique. In this respect Badiou is explicit: "There are not 'several' voids, …" Of the nothingness, being-void is no predictive property and therefore there is no possible plurality of void. The void is unique because "the unicity of the void set is immediate." There is nothing, no predicative value that could differentiate one void from another. Nor is the assignation of common names to the void set enough to overthrow the unique predicative properties of set membership. The entire axiomatics of the ordinal constructibility of sets can be made by founding "*difference on something other than belonging*." A plurality of voids is inextensible because the principle of extensionality is given as predicative equivalence, so any one "void" is ontologically indistinguishable from another. Any differential consideration of void within Badiou's strict set-theoretical understanding of ontology would require entirely new, non-normal principles that would enable the One to *be*, and not to be *not*: "*it is because the one is not that the void is unique*." (Badiou 2005, 68-69) In a well-founded ontology the void-as-nothing is unique.

Yet the proper name of being $\varnothing$ and the "one that is not," may not be existentially equivalent. There may be an ontological non-equivalence between the void set $\varnothing$ and what Badiou refers to in *Logics of Worlds* as the proper inexistent $\varnothing_A$ of a multiple being A. On the one hand, the empty set $\varnothing$ is ontologically void, while on the other hand $\varnothing_A$ is ontologically full. This entails the ontological proposition $\varnothing \neq \varnothing_A$. By the axioms of foundation and extensionality, the referent of the proper name of being $\varnothing$ is the empty set ontologically exists and is unique. However, the proper inexistent of the multiple A is simply the proper class $\{A\} = \varnothing_A$, which is ontologically non-well-founded. In a consistent ontology, non-well-founded sets simply do not exist. In this sense non-well-founded sets are legislatively void. However, the occurrence of the ontological anomalies of proper inexistents and non-well-founded sets constitutes the ontological singularity of an event.

The notion that ontological nothingness may simultaneously be both empty and full, leads to the idea that a world in which an event momentarily occurs, becomes ontologically ill-founded. By admitting the singularity of a non-well-founded situation – an evental site – the world experiences the disruption of being ill-founded. In ill-founded worlds a plurality of void objects may arise in at least two distinct ways. The "two voids" $\varnothing$ and $\varnothing_A$ are diametrically locatable. If, as we shall see, the question of presentation and representation of void following the occurrence of a singularity is one of *copresentation* and *corepresentation*, then the co-suture-to-being emerges out of the coexistence of a singularity whose co-appearance in a world turns on a contingency. I will this cuss this in Chapter Two and Chapter Three.

We will treat a situation immediately prior to an event as a pre-site, which is no longer made up of the discrete multiples the ontological situation but turns into a smooth geometric algebra, of open sets and covers at the moment of the event. We can now say that the greatest and least fixed points of the situation is the union of every subset $U \subset A$ of open sets and covers indexed to the positive real numbers. The greatest fixed point (g.f.p.), which we mark by the sign "**1**," and the least fixed point (l.f.p), which we mark

"$\mathbf{0}$," close the non-infinite closed interval $[\mathbf{0}, \mathbf{1}]$. Note that the real interval $(\mathbf{0}, \mathbf{1})$ is infinite whereas the closed real interval $[\mathbf{0}, \mathbf{1}]$ is not.

Rather than continue in defiance of the axioms of set theory, to speculate the possible ontological existence of two voids, there is a sense in which an equivalence between both void objects, $\varnothing$ and $\{A\} = \varnothing_A$, holds under the name "nothingness." We shall see in Chapters Two and Three that this equivalence is a bisimulation given as a proof technique for the trace equivalences of mutating automata. Such automata, as we shall see, are abstract pseudo-dialectic naming bots, which navigate dynamic traces that resist the flows of historicity. As the force of historicity swells to prevent the formation of a subject of a singularity, a mutant automaton that emerges from what I refer to as the terminal singularity of the pseudo-dialectic turn. Such automata invade the registers of the imaginary while simultaneously maintaining the mutant integrity of the dynamic trace I call *sinthôme*.

Representation

Badiou draws a distinction between the normal ordinal count of a situation – which is determined inductively – and the cardinal count of the *state* of the situation. The cardinal count of a situation is determined by the power set axiom applied to its ordinal count. The power of the state of representation of a finite situation is its cardinal count. As we have seen, this is always greater than its ordinal count by a power factor of two. However, for the purposes of this discussion, we will call the set of *presented* multiples that make up a situation, the *state of presentation* of the situation, whose *count* is its ordinal count. The set of the (ordinal) states that make up the set of all subsets – the power set of the situation – is called the *state of representation* of the situation, as it is in Badiou's writings.

For a situation A, with subset $B \subseteq A$ whose elements are indexed to the ordinal numbers $n, m \ldots \in \mathbb{N}$ (where "$\mathbb{N}$" denotes the set of natural numbers), the number of possible states of representation of B is 2^n, this being the cardinal measure of the power set $\mathcal{P}(B)$. If A has m members then A has 2^m possible states of representation. The *cardinal* measure of the states that represent multiples forms a second meta-structural count (Badiou 2005, 83-84), which is always in excess of the ordinal count of the elements a multiple presents in a situation. The classical logic of Badiou's set-theoretical ontology, is inductively determined and closed under the axioms of Zermelo-Fraenkel set theory. The greatest finite ordinal count of any situation is always less than ω, except when, under certain conditions, the situation becomes a site and transfinite measures $\omega_{\alpha \leq 0}$ can be counted.

Clearly, the cardinal inequality $|\varnothing \subseteq A| \neq |\varnothing_A|$ is true. By the power set axiom, $\varnothing$ is a part of the multiple A (denoted $\varnothing \subseteq A$), but the cardinal measure of a subset of a multiple is always less than the cardinal measure of the set itself. The representational measure of the empty set, which by the power set axiom is a part of the multiple A, is absolutely different from the cardinal measure $2^{\varnothing_A}$ of the proper class $\{A\}$. However, an equality relationship between the two ontological "void" objects, the initializing void $\varnothing$ and the terminal object $A = \{A\}$, holds when their respective, absolutely cardinal divergent states of representation are algebraically located at the same fixed point. We shall see the post-evental equality between the "voids" is maintained by the bisimilarity of their trace equivalences. Similarly, any relationship that survives the event may hold under the cotranscendental ordering of its world. This point is also the exact topological location where the name "void" holds true. That point may be either the greatest or least fixed point of the pre-site order I call a B-transcendental order of the situation. As we shall see, under certain conditions a *B*-transcendental order [1] can be expressed as an atomless Boolean algebra.

In *Logics of Worlds*, the idea of a material onward dialectic flow – whose material sublation occurs at the precise moment of the event – is conceptually incorporated into the ontological induction mechanism that characterizes the count of a situation. Under an evental configuration, the count of a situation *A* counts itself as part of the count. In an event, the resulting singularity $\varnothing_A$ becomes absolutely evident. The evental termination of the count, the closure of the hybrid inductive *aufheben* of Badiou's materialist dialectic, occurs at exactly the *greatest fixed point* of the *B*-transcendental order of a situation.

Infinity is Real

The set of ordinal numbers $\mathbb{N}$ is a proper subset of the reals numbers $\mathbb{R}$, thus $\mathbb{N} \subset \mathbb{R}$. We also know that the cardinal measure of the ordinals is less than the reals. Thus $|\mathbb{N}| < |\mathbb{R}|$.

To each natural number there is a set that contains them all. The set of all natural number is usually denoted $\mathbb{N}$ but set theorists refer to this set as ω. The set of all ordinal numbers is the successive ordering of the naturals.

The ordinal count of a finite situation n is also finite, where $n = \{0, 1, 2, \ldots, n-1\}$ is inductively constructed from the empty set $\varnothing = 0$, $\{\varnothing\} = 1$, $\{\varnothing, \{\varnothing\}\} = 2$, and so on. This count becomes infinite when the situation becomes a site and its ordinal measure is greater or equal to ω_0 and its cardinal measure is $\mathbb{R}$. It is plausible that $\omega_1 = |\mathbb{R}|$. (Easwaran 2005a, 6) The elements of a situation can be put into a canonical relationship

[1] Technically a *B*-transcendental order is a partial order of the sets that make up the situation. The term "*B*-transcendental" plays on the idea of a classical Boolean partial order of a situation, as opposed to Badiou's intuitionistic Heyting-algebraic ordering of a world he calls a "transcendental" order. Later, I will introduce the notion of the coalgebraic dual of a transcendental order. I call this a "cotranscendental" order.

with the natural numbers and the parts of a situation can be canonically represented by the cardinal count of the situation. The count of any non-site situation is always finite while the count of a site is momentarily infinite.

A situation S is a collection of presented multiples, whose power of representation is itself *represented* by its power set, denoted $\mathcal{P}(S)$. An equivalence relation between two situations S and S', which are considered to have exactly the same multiples, can be given by a bijection $f : S \rightarrow S'$ if there is a two-way (onto and into) reciprocal mapping corresponding to each presented multiple in each situation. By the power set axiom, the cardinal measures of S and S' are equivalent if and only if there is a bijection between them. The power of representation of one situation is exactly the same as that of another situation when there is a bijection between them. When this is the case we can say $|S| = |S'|$, which denotes a representational equivalence between states.

It can also be shown that there is no bijection between ω and the set of real numbers $\mathbb{R}$. This means that we can unequivocally refer to the power set $\mathcal{P}(\omega)$ as $\mathbb{R}$ because the cardinal measure of a multiple is always less than the cardinal measure of its power set. This also means that the measure of an evental site can be counted against the smooth space of real numbers.

If a situation is purely ontological, and its ordinal count is extended beyond its largest finite count ω_0, certain conditions can prevail. Paul Cohen showed that Cantor's transfinite continuum hypothesis and the axiom of choice are both independent of the other axioms and rules of classical set theory. Under certain conditions, which turn out to be the elements of a B-transcendental, it can be shown that questions of discernibility and decidability relating to the transfinite ordinal and cardinal count of a situation, cannot be settled under the normal principles of axiomatic induction. For example if κ is a transfinite cardinal measure of a multiple included in a situation, if $\kappa < 2^{\omega}$ then $2^{\kappa} = 2^{\omega}$, but only when $\kappa = \omega$. This is ontologically significant if there is some transfinite indiscernible κ such that $\omega < \kappa < 2^{\omega}$. The only reason κ is "indiscernible," is because any question relating to its statist measure cannot be settled under the *normal* ontological conditions that determine the situation. However, there are non-normal conditions under which certain ontological questions relating to indiscernible transfinite multiples can be settled. In a non-normal situation, which happens to be a site, something extraordinary may happen: a subject can name a radically indiscernible material multiple, in ways which force its transfinite materiality, its infinite being-as-being, to be named **1** (in a bold typeface), which is not **One**. The proper name of being, **One**, can be easily misunderstood if it's purely ontological name **1**, is confused with the ordinal referent of the numeral 1. **1** marks the upper-bound edge of ontological reason. The ontological *conditions* of the occurrence of **One**, under Badiou's genius recognition, happen to be the forcing conditions which American mathematician Paul Cohen first introduced in his seminal 1963 paper "The Independence of the Continuum Hypothesis" (Cohen 1963, 1964).

Although the question of the existential significance of the evental appearance of pure being is fully explored in *Logics of Worlds*, the notion of an abstract subject, who "realizes an indiscernible" is virtually abandoned. However, the idea of an abstract subject, which I define as a mutant automaton, lies at the heart of our exploration of the mutant trace I have named *sinthôme*. In this respect, the seminal work of Badiou in *Being and Event* is significant, especially as we explore a Cohen motivated ontology in which names and a naming function play a meaningful role. This exploration, although horrendously formal, involves setting up an ontological situation in which the forcing principles of Cohen, which are the core principles of Badiou's early ontology of the event, are given a modern treatment. This treatment employs algebraic abstractions of a *separative dense atomless Boolean algebra* with *ultrafilters*.

Forcing

Since the being-there of a world is a function of appearing in a world, the function of appearing is simply a *relationship* between the cardinal measure (i.e. ontological power) of a multiple and its existential coefficient of appearing. In other words, the ontological cardinal "value" of an object is related to its algebraic coefficient of appearance. The later is given by the degree of appearance of the object, which ranges over a real interval closed under maximal and minimum limits. If in the moment of the event, the cardinal measure of this closing is extremely high or too large, as it is with the appearance of a singularity, then "applying a powerful operation to a small world, … , will easily overstep the world." (Badiou 2009a, 331) In closed mathematical systems, such as the complete Heyting algebra that underlies the existential indexing of world and the complete set theories that register ontological "thought" certain limits are encountered. As Badiou points out: "it is impossible for a being to be of a [cardinal] magnitude equal to the world itself. … No being-there of [a world] **m** has the same cardinality as **m** itself." (ibid. 333) This implies that the magnitude of a world is intrinsically greater than that of the multiple-beings which ontologically compose it. This is, of course, is a reiteration of the proof that the cardinal measure of a multiple is intrinsically larger than its ordinal measure. If the 'maximum' ordinal measure of a world marks the upper-bound ontological closure of that world – let the sign of the greatest countable natural number $\mathbb{N}$, mark the upper-bound closure of that world – then the cardinal measure $|\mathbb{N}|$[2] of the states (subsets) of the world is in excess of $\mathbb{N}$ because the cardinal measure of a set is always greater than the ordinal count of its elements. This makes $|\mathbb{N}|$ an "inaccessible infinite" and "even the "smallest" of infinities ω_0 is not accessible." (Badiou 2009a, 334, 335)

In the universe of sets, every set has a power set which is the set of all subsets of itself. If two sets share a bijective relationship with each other, then they have the same

[2] Presume the cardinal measure of $\mathbb{N}$ is equivalent to that of ω_0. In symbols, $|\mathbb{N}| = |\omega_0|$.

cardinality, e.g. $|A| = |B|$. A relationship between sets is *bijective* if there is a one-to-one correspondence between the members of one set and the members of the other.

One treatment of Cantor's transfinite continuum hypothesis (CH) (which we shall discuss in depth later) states that any infinite subset of a real line interval of the set of real numbers $\mathbb{R}$, has either the same cardinality as the (infinite) real line itself, or it is denumerable (or uncountable). That is, for some infinite $B \subseteq \mathbb{R}$, either $|B| = \mathbb{R}$ or $|B| = \mathbb{N}$. Gödel demonstrated that the continuum hypothesis does not contradict the axioms of Zermelo-Fraenkel set theory, but it was not certain if the continuum hypothesis could be derived from the axioms (Gödel 1958). This uncertainty does not prove that transfinite sets do not exist. If transfinite sets do exist it is plausible that they are indeed indiscernible. This idea informs Badiou's early mathematical treatment of the event. The generic forcing procedures of the "Matheme of the Indiscernible" and the significance of the "Cohen Event" is seminal in the history of mathematics (Badiou 2005, Part VII-PartVIII).

In 1963, the American mathematician Paul Cohen (1934-2007) proved that the continuum hypothesis is independent of the axioms of set theory (Cohen 1963, 1964). Cohen also proved the independence of the axiom of choice. Although Cohen claimed that the continuum hypothesis is patently "false,"[3] this does not mean that infinite and transfinite

[3] Paul Cohen: "... CH is obviously false. The main reason one accepts the axiom of infinity is probably that we feel it absurd to think that the process of adding only one set at a time can exhaust the entire universe. Similarly with the higher axioms of infinity. Now $\aleph_1$ is the cardinality of the set of countable ordinals [our equivalent to $\mathbb{N}$], and this is merely a special and the simplest way of generating a higher cardinal. The set C [CH] is, in contrast, generated by a totally new and more powerful principle, namely the power set axiom. It is unreasonable to expect that any description of a larger cardinal which attempts to build up that cardinal from ideas deriving from the replacement axiom can ever reach C. Thus C is greater than $\aleph_1, \aleph_\omega, \aleph_a$, where $a = \aleph_\omega$, etc. This point of view regards C as an incredibly rich set given to us by one bold new axiom, which can never be approached by any piecemeal process of construction. Perhaps later generations will see the problem more clearly and express themselves more eloquently." (Cohen 1966, 151)

indiscernible objects and their measures do not exist. The continuum hypothesis inductively postulates that the inductive ordinal structure of transfinite objects continues beyond the greatest discernible object ω_{0-1}. Badiou's early notion that the power of a state of representation is always greater than the power of presentation in the universe of sets, entails the idea that the (cardinal) power-set measure of a multiple is always in excess of the natural ordinal count of its presentation. This is especially true with transfinite multiples. Cohen's method is to "force" a transfinite object greater than the least transfinite number ω_0 into the known universe of sets. This new object, which is ontologically indiscernible, expands the *original* universe of sets into a new larger one. The resulting new universe may have many more new subsets of ω that were not in the unexpanded universe.

An approach to Cohen forcing, which follows American mathematician Kenneth Kunen (b. 1943), who following Solovoy and Scott (Fourman et al. 1979; Scott 1960, 1961), presented an expanding universe approach to Cohen forcing showing. This method demonstrates how unramified forcing over the complete models of Zermelo-Fraenkel set theory can be rephrased as Boolean-valued models (Kunen 1980). In the Nineteen Seventies, American philosopher and logician Saul Kripke (b. 1940) noticed striking similarities between modal logic, intuitionistic logic and Cohen forcing. As we shall see, all modal logics are coalgebras and intuitionistic logics can form the logical models for a topological open-set ontology of an evental site. Unramified forcing techniques help develop an elementary ontological model of a situation, under which the conceptual passage from algebraic indiscernible to coalgebraic namable is plausible.

When a situation is generically extended, the names of any state (or multiple), both finite and infinite, are accessible to any linguistic subject inhabiting that situation. However, only the finite states are accessible to a subject. The axioms of Zermelo-Fraenkel set theory, together with any well-formed sentence validly derived from the axioms, constitute an ontological language without names for transfinite situation (the set of all ontological sentences without names that apply to a situation is, Badiou's nomenclature, "encyclopedia of the situation" (Badiou 2005, 328, 329)). Such a language describes finite states, the idea that there is at least one infinite set is entailed by the axiom of infinity (which is not derivable from the other axiom of ZF). From the axiom of infinity we postualte that infinity exists, namely the set of natural numbers, but any speculation about infinite states whose count is greater than ω_0, is independent from the axioms of set theory. On the other hand, subjectivizable, forcing languages have names for indiscernible objects. This is because there exists a number of "subjectivizing" functions or operations for finding names of indiscernible ontological entities that somehow bear truth.

As our discussion of forcing expands, we begin to adopt a more contemporary, algebraic formalization of Cohen forcing. This algebraic treatment of ontology is more compatible with the specific coalgebraic matheme of the indiscernible, which will be introduced in Chapters Two and Three. A categorial approach to this development allows an easier theoretical transitions between the notions of Cohen forcing, indiscernible objects, singularities and the event. A topology of the body that is purely coalgebraic, is categorially easier to theoretically develop than a classical set-theoretical approach would otherwise allow. As we shall see, a categorial approach to coalgebra and functor categories provides a rich theoretical environment and powerful mathematical tools with which to develop a formal description of a mutant automaton and the pseudo-dialectic turn and its corresponding trace, the *sinthôme*.

Categorial Forcing

We have seen that the cardinal measure of the set of real numbers is equal to the power set of the natural numbers, i.e. $|\mathrm{R}| = \mathcal{P}(\mathbb{N})$. Monic arrows in category theory, are the categorial equivalent to set injections, or "into" set functions, while epic arrows (or epimorphisms) are the categorial equivalent to set-theoretical surjections, or "onto" functions.[4] Categorially speaking, Cohen's problem was to find a object B (in the category **Set**) in which no two monic arrows exist that give the injective functions $\mathbb{N} \rightarrowtail B \rightarrowtail \mathcal{P}(\mathbb{N})$ such that there are no two surjections $\mathbb{N} \twoheadrightarrow B$ and $B \twoheadrightarrow \mathcal{P}(\mathbb{N})$. Cohen took a knowable universe of sets and then expanded it by *forcing* a new set B of subsets of $\mathbb{N}$ into this new expanded universe in such a way that the cardinal measure $|B|$ lies is strictly between the cardinal measure of $\mathbb{N}$ and the measure of its power set, that is $|\mathbb{N}| < |B| < |\mathcal{P}(\mathbb{N})|$.

No set can be put into a bijection with its power set. However, the power set of the smallest infinite set ω_0 can be put into a bijection with the set of real numbers $\mathbb{R}$. Cantor's diagonal argument shows $\mathbb{R}$ is not denumerable. Putting this in the language of Badiou, not all reals are discernible. It follows that if some reals are indiscernible, in the sense of being uncountable, then this is either because they are too large or two small. By definition the set of natural numbers $\mathbb{N}$ is discernible. However, given $\mathbb{N}$, then by the principle of cardinality, $\mathbb{R}$ can be identified, with the power set of $\mathcal{P}(\mathbb{N})$, which is ω_0 under another name. Because the diagonal argument shows that any map $\mathbb{N} \xrightarrow{v} \mathcal{P}(\mathbb{N})$ is not bijective, then $\mathbb{N} < \mathcal{P}(\mathbb{N})$. If $\xrightarrow{v}$ is a bijection then the measure of $\mathbb{N}$ would be equal to the measure of its power set, i.e. $\mathbb{N} = \mathcal{P}(\mathbb{N})$.

---

[4] A monic arrow, or monomorphism $f : a \rightarrowtail b$ is the categorial equivalent of the injective (one-one) set function $f : A \rightarrow B$. A set function is injective when no two set distinct inputs give the same set output. For inputs $x, y \in A$, if $f(x) = f(y)$, then $x = y$. An arrow $f : a \rightarrow b$ in a category $\mathcal{C}$ is monic in $\mathcal{C}$ if for any parallel pair $g, h : c \rightrightarrows a$ of $\mathcal{C}$-arrows, the equality of the compositions $f \circ g = f \circ h$ implies that $g = h$. (Goldblatt 1979a, 37-38).

A set function $f : A \rightarrow B$ is onto, or surjective, if the codomain B is the range of f. Similarly, an epic arrow, or epimorhism (i.e. a surjective homomorphism) is a $\mathcal{C}$-arrow $f : a \twoheadrightarrow b$, which holds if for any parallel pair $g, h : b \rightrightarrows c$ of $\mathcal{C}$-arrows the compositional equality $g \circ f = h \circ f$ implies that $g = h$.

The continuum hypothesis asserts that there is no set (or situation) S such that $\mathbb{N} < S < \mathcal{P}(\mathbb{N})$. This implies that every infinite set in $\mathbb{R}$ is either denumerable or it has the same cardinal measure as $\mathbb{R}$. The evental forcing of an indiscernible violates this principle by taking a complete, countable transitive model (a quasi-complete situation) of set theory and including in it as a subset, some set B which is larger than the power set of $\mathbb{N}$. Because, by definition, S is denumerable, then the count of S is either equal to, or less than, the count of $\mathbb{N}$. Let B, which is larger than $\mathcal{P}(\mathbb{N})$, be the next power set $\mathcal{P}(\mathcal{P}(\mathbb{N}))$. If we construct a new model M' which extends M and treat it as a category of **Set** containing the monic arrow

(i) $\quad g : B \rightarrowtail \mathcal{P}(\mathbb{N})$

then $\mathbb{N} < gB < \mathcal{P}(\mathbb{N})$ holds for all corresponding cardinal measures in M'. Since $\mathcal{P}(\mathbb{N}) = 2^{\mathbb{N}}$ we can replace g in (i) above, with its transpose $f : B \times \mathbb{N} \rightarrow \mathbf{2}$ where for some $b \in B$ and $n \in \mathbb{N}$, the output of $f(b, n)$ is either equal to $\mathbf{0}$ or $\mathbf{1}$, according to whether either $n \in g(b)$ or $n \notin g(b)$ (remember, the set $\{\varnothing, \{\varnothing\}\} = \{0, 1\}$ is a primitive way of denoting 2, but the set $\{\mathbf{0}, \mathbf{1}\} = \mathbf{2}$ marks the lower and upper-bound limits of our B-transcendental).

The function $f : B \rightarrow \mathbb{N}$ in $B \in \mathbb{N}$ forms a many-valued list of all the ordered pairs $\langle b, n \rangle$ with $n \in g(b)$. For g to be a monic arrow on b, then some b' must also be an element of B. $b \neq b'$ implies that $f(b, n) \neq f(b', n)$ for some n. However, according to the diagonal argument there is no such function f in any given model S. Even though there is no such function f in S, there are finite approximations to it. Such a finite approximation is a finite subset $F_p : B \times \mathbb{N}$ and the function $p : F_p \rightarrow \mathbf{2}$.

Such a (F_p, p) is often called a *forcing condition* of p. A forcing condition is just two disjoint lists of ordered pairs $\langle b_i, n_i \rangle$, $\langle c_i, m_i \rangle$ of elements of $B \times \mathbb{N}$ with $p(b_i, n_i) = 0$ and $p(c_i, m_i) = 1$.

These conditions constitute the partially ordered set $\mathbb{P}$, with the partial order defined by $q \leq p$ iff $F_q \supseteq F_p$ with q restricted to F_p. The later coincides with p. If $p \leq q$ then q is a closer approximation than p is to the function f, i.e. q gives more information about f and q . When this is the case we can say that is an "extension" of $\mathbb{P}$.

The partial order $\mathbb{P} = \langle P, \leq \rangle$ is often referred to as the *notions of forcing*, while the elements of $\mathbb{P}$ are its *forcing conditions*.

The above treatment of Cohen forcing is primarily due to the powerful categorial description (with slight modification) of Cohen topoi by American mathematician Saunders Mac Lean (1909-2005) in (Mac Lane and Moerdijk 1992, 277-278).

Forcing a *B*-transcendental

Badiou's schematization of the "transcendental" of a world is an intuitionistic algebraic ordering of the multiples that constitute a world. Technically, a transcendental is a partially ordered set. My description of the forcing of transfinite indiscernible, involves ordering a situation as the classical Boolean valued partial order I call a *B*-transcendental. The ontological structure of any situation may be given as a partially ordered set of multiples. A partially ordered set is often called a *poset* in the literature. Care must be taken not to confuse the existential efficacy of Badiou's notion of a transcendental partial ordering of a world with my purely ontological description of a partial order of the multiples that make up a situation. The site of an event, as it is described in *Logics of Worlds*, is a smooth topological space. A Badiouian transcendental describes the existential character of a world, whereas a *B*-transcendental describes the ontological order of a situation and a site. In *Being and Event* a site is schematized as a generically extended set of discrete multiples, some of which have transfinite values and are therefore ontologically indiscernible. My idea of a *B*-transcendental follows a modern approach to mathematical forcing by describing a countable transitive model of Zermelo-Fraenkel set theory as a Boolean partial order. I will later describe the *cotranscendental* order of a world that has fallen under a pseudo-dialectic turn of events. A pseudo-dialectic turn entails the occurrence of a *silent singularity*, which is a type of pseudo-event. A silent singularity is mathematically equivalent to what Badiou refers to as a "weak singularity." (Badiou 2009, 389-396) Because their transcendental appearance is minimal, weak singularities form evental sites but do not constitute an event. As we shall see, silent singularities are the initial objects of the mutant trace that is *sinthôme*.

Let a pre-site situation be the smallest partial order of the situation. Call this partial order a *B*-transcendental. Badiou calls an ontological situation so ordered, a *quasi complete situation* (Badiou 2005, 358). The ontological set of Cohen forcing conditions that presage the evental naming of an indiscernible, is a partial order $\langle \mathbb{P}, \leq \rangle$ such that the elements of $\mathbb{P}$ are called forcing conditions. These can recursively define the subjectivizable object, which we shall call a $\mathbb{P}$-*name*. [5] As we shall see, such an object is a generic *name* of some transfinite indiscernible part of the *B*-transcendental $\langle \mathbb{P}, \leq \rangle$. The inclusion of a $\mathbb{P}$-*name* in certain generically extended Boolean models of set theory, whereby $\mathbb{P}$-*names* can be used to describe the indiscernible objects of evental situations (sites).

Ontologically speaking, all pre-sites are partially ordered sets, or posets. A poset $\langle \mathbb{P}, \leq \rangle$ is a set of multiples $\mathbb{P}$ with a binary relation $\leq$ such that for any two multiples $x, y \in \mathbb{P}$, the relation $x \leq y$ is reflexive, antisymmetric and transitive.[6] Commutativity, associativity, distributivity and negation also hold. If we consider an ontological situation to be a Boolean algebra of sets, not merely a classical set-theoretical construct of first-

---

[5] The term $\mathbb{P}$-*name* is due to (Kunen 1980, 52)

[6] The relation $\leq$ over elements $x, y \in \mathbb{P}$ is (i) *reflexive* when $x \leq x$, (ii) *antisymmetric* if $x \leq y$ and $y \leq x$ then $x = y$ and (iii) *transitive* if $x \leq y$ and $x \leq y$ then $x \leq y$. A *total* or linear ordering, is a partial ordering with the additional requirement of *trichotomy*, i.e. for any $x, y \in \mathbb{P}$, either $x \leq y$ or $y \leq x$ or $x = y$.

order logical sentences, then this situation is classical inasmuch as the double negation also $\neg\neg x \leq x$ holds. As we shall see, the logical integrity of double negation does hold in a transcendental order of a world.

A Boolean algebra can also be thought of as a particular kind of *B*-transcendental which must have minimal and maximal elements, usually denoted "**0**" and "**1**" respectively. In addition, every pair of elements x, y must have a least upper bound $x \sqcap y$ (infimum) and a greatest lower bound $x \sqcup y$ (supremum). The *B*-transcendental order of a situation can also be considered to be a complete lattice of the situation. Occasionally I will use the term "lattice" to denote a conceptual alternative partial order. If A is a subset of a lattice $\mathbf{L} = \langle L, \sqsubseteq \rangle$,[7] then there is an upper bound element x of A, denoted $A \sqsubseteq x$, if $y \sqsubseteq x$ if $y \in A$. If $x \sqsubseteq z$ whenever $A \sqsubseteq z$, then x is a least upper bound of A. A has at most one least upper bound.

An element x is the greatest element of A if x is a least upper bound of A but is also contained in A. Therefor, A has a greatest element when one of its members is a least upper bound of A. The greatest lower bound of A is defined dually. (Goldblatt 1979a, 178-179)

Generic Ultrafilters

[7] I use the sign $\sqsubseteq$ instead of the more common $\leq$ to denote the ordering of a poset. Rather than solely interpreting the partial relationship $\leq$ as "greater or equal to *y*," as we see in of *Logics of Worlds*, the use of the symbol " $\sqsubseteq$ " suggests an intuitive interpretation of: (i) the temporal or modal operation "*x* before, or concurrently with *y*", (ii) "either *x* entails *y* or *x* is equivalent to *y*". Sometimes I will use the sign "$\sqsubseteq$" instead of "$\leq$". This convention follows Rob Goldblatt who often refers to the partial orders of Heyting and Boolean-valued sets to describe modal logical operations (Goldblatt 1979a, 1993).

In an ontological situation with a *B*-transcendental order, the prime ideal and the ultrafilter of the order, isolate the 'lower' and 'upper' edges of the situation. Prime ideals and ultrafilters mark the ontological upper and lower bound edges of reason relative to the void. We generically extend a situation *S* by adding another set ♂ and express the ontological structure of an event as the generic extension of *S* by ♂. Let *S*[♂] denote the generic extension of *S* by ♂. If we ontologically construe the situation *S* to be a Boolean-valued model of axiomatic set theory with a *B*-transcendental order, then the generic multiple ♂ is exactly a Boolean ultrafilter (c.f. (Badiou 2005, 356; Easwaran 2005, 2-4; Kunen 1980, 53, 76)). A Boolean ideal is a non-empty proper subset of a Boolean algebra that is closed downwards under $\sqcup$. Generally, this means that if we let *I* denote an ideal and let *F* denote a filter such that if $x, y \in I$ and $z \leq x$, then $z \in I$ and $x \sqcup y \in I$. Similarly for a filter *F*, if $x, y \in F$ and $z \leq x$, then $z \in F$ and $x \sqcap y \in F$.

If *I* is an ideal and *F* is a filter, then the set $\{x : \neg x \in I\}$ is a filter and the set is a $\{x : \neg x \in F\}$ ideal.

Boolean algebras can be also be thought of as binary sets of possible truth values, which can be assigned to the sentences of formal logical languages. Such sentences include those which describe ontological situations and determine their count. One possible axiological 'count' of such sentences, may therefore be just the binary values **0** and **1**. If the set of those sentences which are deemed to be true are named **1** then the specified set of such sentences is an ultrafilter. Dually, where the set of those sentences which are deemed to be false are named **0**, then the specified set of such sentences is an ideal.

If one distinct Boolean algebra is mapped to another Boolean algebra by some function in such a way that all the operations $\sqcap, \sqcup, \neg$ and associativity, etc. over the variables of *A* and *B* are preserved, then the set of elements mapped to **1** are filters and the set of elements mapped to **0** are ideals. This idea is important because it involves the "naming" of an indiscernible by determining its truth value "located" in the generic ultrafilter ♂. In a Boolean ontological situation, the ontological 'count' of truth will always be always **1**, while the proper name of truth can be expressed as '**One**.' **One**, can be validly assigned to an (transfinite) indiscernible whose ontological ordinal measure is greater than ω. However, the Boolean count of **One** can be no greater than **1**. **1** marks the supremum of any *B*-transcendental. The supremum of any situation marks the upper bound edge of all that is discernible.

Boolean Situations and an Atomless *B*-Transcendental

Let a situation be Boolean if its *B*-transcendental is ontologically describable as a complete atomless Boolean algebra with dense subsets. The following describes such an ontology.

Let S denote a Boolean situation. For any state (or multiple) $x, y \in S$, let the relation $x \leq y$ mean $x \subseteq y$, then clearly, set theory is also a Boolean algebra with $x \sqcap y = x \cap y$, $x \sqcup y = x \cup y$ and $\neg x = S \setminus x$. [8] This new set-theoretical algebra is a *complete* Boolean algebra if every set of elements has a least upper bound (their union or "envelope") and a greatest lower bound (their intersection or "conjunction"). In this algebra, we will consider the atoms of a situation to be the singleton sets $\{a\}$, where every $a \in S$. This means that if $x \leq \{a\}$, then either $x = 0$ or $x = \{a\}$. A Boolean algebra is *atomic* if every element has an atom below it. By the axiom of foundation, the algebra of the situation S is "well-founded." An algebra is *atomless* if it is without atoms. Of course, this notion violates the axiom of foundation, but the idea of an atomless Boolean algebra is consistent with forcing over Boolean-valued sets and the coalgebraic treatment of post-evental situations.

---

[8] The complement $S \setminus x$ expresses the idea that $\neg x$ is S without x.

Consider a Boolean algebra B to be the partially ordered situation $B \setminus \{\mathbf{0}\}$.[9] This means that B is atomless (B without $\{\mathbf{0}\}$). We define an atomless partial order $\forall x \exists y\ (y < x)$[10] when there is no z such that $z \leq x$ and $z \leq y$. In this partial order we can define the relation $z \perp y$ [11] if and only if x and y have no common lower bound.

A situation $S = \langle \mathbb{P}, \leq \rangle$ is separative if the following conditions hold

(i) $x \not\geq$ y means $x \sqcap y = y$

(ii) since $y = (y \sqcap x) \sqcup (y \sqcap \neg x)$ by propositional logic, then $y \sqcap x \neq 0$

(iii) if $z = (y \sqcap \neg x)$ and $x \not\geq y$, then $\exists z\ (z < y$ and $z \perp x)$

Martin's Axiom

---

[9] "$B \setminus \{\mathbf{0}\}$" denotes the relative complement of the Boolean algebra B and the singleton $\{0\}$. , i.e., B *is* without $\{\mathbf{0}\}$ making it "top open" under $\sqcup$. In classical set theory, the set $\{0\} = 1$.

[10] The sign "$<$ " signify the less-than relation $\langle y, x \rangle \in <$.

[11] The sign "$\perp$" is often called the *falsum*, meaning the "false," or a logical contradiction. Its dual, "$\top$," signifies the *tee* or the "true," or a logical tautology. In our context, $\top$ and $\perp$ relate to **1** and **0** respectively. However, the signs $\perp$ and $\top$ are often used in conjunction with "$\vdash$", the turnstile, meaning "yields to" or "proves." If we have a complete model M of ZFC set theory, then the sentence M f, or alternatively, $\phi \vdash_M \top$ means ϕ is derivable (is true) in M.

Martin's Axiom (abbreviated MA)[12] was introduced by American mathematicians Donald Martin and Robert Solovay in 1970 (Martin 1970). Martin's Axiom, also known as the *proper forcing axiom*, is a statement about transfinite cardinal values and the independence of the continuum hypothesis from the axioms of Zermelo-Fraenkel set theory.

The generalized form of Martin's Axiom is implied by the continuum hypothesis, so it is consistent with ZFC (the axioms of Zermelo-Fraenkel plus the axiom of choice), but it is also consistent with ZF + $\neg$ CH. This implies that, in ontological situations, Martin's

---

[12] Martin's Axiom as stated in (Martin 1970, 144):

> We introduce an "axiom" A which (1) is demonstrably consistent with ZF, (2) allows the continuum to be (loosely speaking) any regular cardinal, (3) follows from CH and implies many of the important consequences of CH, and (4) implies, when $2^{\aleph_0} > \aleph_1$, several interesting statements. The following theorem gives some of the main consequences of A. (For a statement of A, see §1.2.)
>
> Theorem. *If* A *then*
>
> 1) $2^{\aleph_0} > \aleph_1$ Souslin's hypothesis […] ;
> 2) If $\aleph$ is an infinite cardinal number $< 2^{\aleph_0}$, then $2^{\aleph} = 2^{\aleph_0}$;
> 3) If $2^{\aleph_0} > \aleph_1$, every set of real numbers of cardinality $\aleph_1$ is $\Pi^1{}_1$ if and only if every union $\aleph_1$ Borel sets is $\Sigma^1{}_2$ if and only if there is a real t *with* $\aleph_1^{L[t]} = \aleph_1$;
> 4) The union of $< 2^{\aleph_0}$ sets of reals of Lebesgue measure zero (respectively, of the first category) is of Lebesgue measure zero (of the first category);
> 5) If $2^{\aleph_0} > \aleph_1$, every $\Sigma^1{}_2$ set of reals is Lebesgue measurable and has the Baire property;
> 6) $2^{\aleph_0}$ is not a real valued measurable cardinal …

§1.2. (Martin 1970, 148):

> The model $\mathcal{M}$ is involved in the notion of an $\mathcal{M}$-generic filter on P only via the collection of dense open subsets of P belonging to $\mathcal{M}$. Accordingly we introduce a more general notion. If P is a partial ordering and $\mathcal{F}$ is a collection of dense open subsets of P, an *$\mathcal{M}$-generic filter on P* is a subset G of P satisfying a) and b) in the definition $\mathcal{M}$-generic filters and
>
> c') $X \in \mathcal{F} \rightarrow X \cap G \neq \phi$.

Axiom is consistent with the axioms of ZF without the axiom of choice. This means that any ontological representation of the continuum hypothesis and its parts, is not properly discernible. However, Martin's Axiom can entail procedures which allow representations that *approximate* transfinite indiscernibles. Therefore, Martin's Axiom, and its various forms, become of particular interest when discussing the ontological make up of an event. This is especially relevant when the ontological *truths* that emerge from a site, cannot be consistently derived from the ontology (axioms) of its corresponding situation. Interestingly, both Martin and Solovay, whom together with "many set theorists," "suspect that CH is false" (Martin 1970, 143).

One of the two main forms of Martin's Axiom is MA(κ). MA(κ) holds where κ is either the finite cardinal measure of a situation, i.e. κ is less than ω, or κ is transfinite cardinal such that

$\omega \leq \kappa \leq 2^{\omega}$. MA($\kappa$) states that for any partial order, which satisfies a countable chain condition, [13] and for any family of dense subsets in the *B*-transcendental $\mathbb{P}$, a dense subset D of the family which has the cardinal measure $|D|$ no greater than κ, then there is a filter on $\mathbb{P}$ such that union of that filter and the dense subset, is not void.

A situation S is dense if there is a subset $D \subseteq S$ which is also dense. If for states $s, t \in S$ and some subset of states $D \subseteq S$, then D is dense if and only if

(iv) $\forall s \exists t \; (t \in D$ and $s \leq t)$

For some set of states ♂ $\subseteq S$, G is a filter in S if and only if for states $s, t, r \in$ ♂

(v) $\forall s \forall t \exists r \; (r \leq p$ and $r \leq q)$, and

(vi) $\forall s \forall t \; (t \leq s \Rightarrow s \in$ ♂$)$

---

[13] A partially ordered set *A* is said to satisfy the countable chain condition, if every strong antichain (i.e. no two elements have a common lower bound) in *A* is countable.

Let some state κ in a situation S be indiscernible in S. This means that the ordinal count of κ is greater or equal to the limit ordinal of S, i.e. $\kappa \geq \omega$. If S is a non-empty situation, and there is a family $\mathcal{D}$ of $\leq\kappa$ dense subsets of $\mathbb{P}$ (i.e. every state of $\mathcal{D}$ is also dense but is lower in the ordinal order of S than κ, because in the least $\kappa = \omega$), then there is a filter ♂ in $\mathbb{P}$ such that the intersection of the filter ♂ and every dense subset of $D \subseteq \mathcal{D}$ is non-empty. This means ♂ $\cap D \neq 0$. By Martin's Axiom, we know an indiscernible state, indexed to an ordinal, is less than the next indiscernible greater than ω. Thus $\kappa < 2^{\omega}$ (Kunen 1980, 54).

If the intersection of ♂ and some dense state $D \subseteq \mathcal{D} \subseteq S$ is non-void, the statement MA(κ) holds for every $\kappa \leq$ CH. The various forms of Martin's Axiom entail the thought that continuous transfinite induction can extend beyond any multiple, which can be represented in a situation. However, this extension can only be meaningfully approximated in states that have already been formed by the inductive operations that maintain the count of any ontologically determined situation.

Under Martin's Axiom, the following statements hold:

(i) A situation S is *finite* if $|S| < \omega$

(ii) A situation S is *infinite* if $|S| \leq \omega$

(iii) When $|S| \geq \omega$ the situation S is a site

(iv) If S is infinite, any transfinite measure $\omega < |S| < 2^{\omega}$ is indiscernible in S

(v) "Infinite" means "not finite"

(vi) "Uncountable" means "not countable"

(vii) A situation S is countable if $|S| \leq \omega$

The situations we are considering are partially ordered sets of a Boolean algebra $B \setminus \{\mathbf{0}\}$. Let the partial order $\mathbb{P} = \mathcal{P}(S) \setminus \{0\}$, with states $s, t, r \in S$ be such that $s \leq t \Leftrightarrow s \subset t$, then $s \perp t$ if and only if $s \cap t = 0$. Some subset $A \subset \mathbb{P}$ is an *antichain* if and only if its elements are pairwise disjoint (i.e. no two ordered paired elements of $\mathbb{P}$ are equivalent nor do they share the same transpositional order). Thus $\mathbb{P}$ has the *countable chain condition* (ccc), if every antichain in $\mathbb{P}$ is countable if and only if $|S| \leq \omega$.

This allows us to assume that for any Boolean algebra B and for a partial order $\mathbb{P} = |S| \leq \omega$, with the same order as B, then s and t are incompatible if and only if $s \sqcap t = 0$ (C.f. (Kunen 1980, 53).

Now, let $\mathbb{P} = \langle P, \leq \rangle$ be a partial order of states that are exactly the finite partial functions

from ω to $\mathbf{2}$ (i.e. $f : \omega \rightarrow \{\mathbf{0}, \mathbf{1}\}$). If $p \in P$ then

$$\mathbb{P} = \{ p : p \subset \omega \times 2 \text{ and } |p| < \omega \text{ and } p \text{ is a function}\}.$$

If $t \subset s$ then we let $s \leq t$ mean that s *extends* t if and only if the domain of s intersects the domain of t. If this is the case, then the union of s and t become the common extension to them both. If σ is a filter in $\mathbb{P}$, then the dense states in σ are pairwise compatible, i.e if $s \leq t$ then $t \leq s$. If we take some function $f = f_{\sigma} = \bigcup \sigma$ then f_{σ} is a function whose domain is also a subset of ω.

If some state $r \in \mathbb{P}$, then we can think of s as a finite approximation of f. If this is the case, then it is plausible to think that the state s forces the condition $r \subset f$. Forcing is an approximation of the observation that r is a state in σ and σ is a filter in $\mathbb{P}$. If this is the case then the state r is contained in the function f_{σ}. Furthermore we can see that r "represents" the particular functions that are restricted to its domain. If t extends s (i.e. $t \leq s$) then the state t is able to represent more about the function f than s can.

Any element of a situation S can be refined (or extended) to get a further element of the situation. This can be achieved when ♂ is an ultrafilter that intersects the various dense parts of the Boolean situation S. If S is a partial order that is a dense subset of a Boolean algebra B and D is a dense part of S, then there is some other dense subset $D' \subset S$ of the situation such that

$$D' \subset S = \{s \in S : \exists t \, (s \leq t \text{ and } t \in D)\}$$

is also dense in S.

Regarding their dense subsets, the situation S and the Boolean algebra B are ontologically interchangeable. This is because the filter ♂ intersecting every dense part of S corresponds to another filter intersecting every dense subset of B.

Intuitively, forcing the conditions of a situation are the elements of its B-transcendental, i.e. the forcing conditions of S are the elements of $\mathbb{P}$. These conditions give us information about x or some other object directly constructed from x. If we interpret the "greater than or equal to" relation $\leq$ to mean "extends," or "expands," then we can say that the state p *extends* q if and only if $p \leq q$. Beginning with a standard transitive model M of Zermelo-Fraenkel set theory and a partially ordered set P belonging to M, for any two states p_1 and p_2 in P, we say that p_2 extends p_1 if $p_1 \preceq p_2$. [14] Intuitively, this sort of inductive transitivity is closer to the suggestion that the sentence "p extends q" implies "p has more information than q." This ordering of the extension relationship is the reverse of our partial order $\mathbb{P}$. Martin defends this positive ordering by noting "We do this for

[14] Martin and Solovay use the notation "$\preceq$" where I use "$\leq$". Both notations are intuitively equivalent and, in this context, denote the binary relation "extends." However, we use "$p_1 \leq p_2$" as the reverse extensional order "p_1 extends p_2". This is the opposite of Martin and Solovay (Martin 1970, 146). The notation "$\leq$" conforms to the traditional meaning of the algebraic binary relation "greater than or equal to," which is followed by Badiou in *Logics of Worlds*. Kunen and others maintain the standard algebraic notation but give it an extensional meaning ranging over a partial order. I leave the Matin and Solovay notation intact letting "$\preceq$" denote a positive ordering.

historical agreement with Cohen [(Cohen 1966)] and because we, like Cohen, think of "*p extends q*" as meaning "*p* has more information than *q*" or "*p* knows more than *q*" (Martin 1970, 147). However, the idea of the reversal of an algebraic order is fundamental to our notion of a pseudo-dialectic turn and the cotranscendental which reverses a Badiouian transcendental ordering of a world.

The idea that one state inductively *extends* another so that information is be passed "onwardly and upwardly" – as it is with under the logic of the materialist dialectic – between states is reversed with the pseudo-dialectic turn and the recovery of "truth" on the level of the symptom.

Moving downwards in the partial ordering always corresponds to getting more information about a situation, since there is always other possibilities that are being either ruled out or going unrecognized (Easwaran 2005, 4).

The reverse ordering of the *B*-transcendental of a situation can entail the idea of extending or expanding information about the states of a situation. However, the binary relationship under $\leq$ implies that *p* entails *q* and so connects the two states, keeping them unseparated. As we have seen, one state in a situation may extend the ontological information held by another state under its relationship with a third state. For this notion to hold, however, the situation needs to be a *separative* partial order.

Generalized Models and Particular Situations

Each multiple in a situation can be ontologically stated as either an axiom of Zermelo-Fraenkel set theory or a logical sentence that can be consistently derived from the axiom. For example, the first order sentence

$$\forall x \forall y \, (\forall z (z \in x \Leftrightarrow z \in y) \Rightarrow x = y)$$

where "$\in$" and "$=$" denote the primitive predicates of set membership and equality respectively, states that any two multiples x and y, that have exactly the same members, are equal. This is the *Axiom der Bestimmtheit* or the axiom of extensionality or determination, Zermelo's first axiom (Kunen 1980, 10).

Let ϕ be a first order sentence and consider a situation S to be transitive,[15] then there is a corresponding sentence ϕ^S. If ϕ^S is ZFC-consistent then we can say $S \vDash \phi$ denoting that S ontologically "satisfies" or "gives" ϕ if and only if the referent of ϕ can be consistently derived from the axioms of set theory. An ontological situation S can be generalized as a model of set theory M such that $S \subseteq M$ and $M \vDash \phi$. A generalized model of ZFC is a special type of situation, which Badiou calls a quasi-complete situation. We will call a situation S,

- pre-site situation, when $S \subseteq M$ and M is a countable transitive model of ZFC and
- its Boolean correlate is a *dense*, *atomless* Boolean algebra.

We denote $S[♂]$ the generic extension of a situation when

(i) $S \subseteq M$ [16]

(ii) M is a countable transitive model of ZFC

(iii) $S = \{S\}$

(iv) $|S| = \omega_\alpha$, where $\alpha \geq 0$

(v) ♂ is a Boolean ultrafilter

---

[15] As we have seen, a situation S is transitive, if for any two states $s, t \in S$, if $s \in t$ and $t \in S$ then $s \in S$.

[16] Any generalized model of ZFC is either equal to, or can "relativize down" to any situation S, but any arbitrary situation, not equivalent to its generalized model, cannot "relativize up." This is because there may be some $X \subseteq M$ that is not in S, cf. (Kunen 1980, 124).

We will focus on a situation in which a sentence ϕ^S is able to consistently convey something about the situation which it could not otherwise convey if the situation was unextended. The generic extension $S[♂]$ of S will provide the formal mechanism to plausibly express something about S when $|S| = |\{S\}|$.

If a consistent ontological situation S can be generalized as a countable transitive model of ZFC, then $S \vDash$ ZFC. By the Löwenheim-Skolem theorem,[17] we know that S is countable. From this we can construct some other, infinite model S^+ which extends ZFC to infinite domains in such a manner that the schema

$$S^+ \vDash \text{ ZFC} + |\mathbb{R}| = \omega_\alpha$$

signifies the shift from the discrete ordinal measure of a situation to the order of real infinite values. This occurs when a situation becomes a singularity and the measure $|S| = \omega_{\alpha \geq 0}$. When a situation S is infinitely *expanded* under the sign of the event, it becomes the site S^+. In the moment of the event, the discrete finite ordinal composition of the situation is extended beyond the edge of the real, whereby the discrete set-theoretical topology of the site is smoothed under the numerical count of the real, $\mathbb{R}$.

We can construct a countable transitive model that satisfies every sentence that is consistent with S, but show that the meaningful sentence "$\mathbb{R}$ is uncountable" is also true in S^+. Although the "elements" of $\mathbb{R}$ remain infinitely uncountable, and therefore indiscernible, a name can emerge from the fluidity of the event, which stabilizes the

---

[17] Restricting the Löwenheim-Skolem theorem to our discussion, the theorem states: if a countable first-order theory (such as ZFC) has an infinite model, then for every infinite cardinal number κ, there is also a model the of size κ. The theorem implies that the first-order description of ZFC is unable to control the cardinality of any infinite extension of its finite models. An example of a finite model with an infinite model is ZFC + CH. The Löwenheim-Skolem theorem also states that no first-order theory with an infinite model can have exactly one consistent model. The Löwenheim-Skolem theorem does not hold for second order logic and other higher logics.

meaning of $\mathbb{R}$ in S. As we shall see, this will ontologically occur in the intersection of the dense atomic subsets of the situation.

When $S = \{S\}$, i.e. when S becomes a singularity, we also know the infinite measure $|S| = |\{S\}| = \mathbb{R}$. However, $\mathbb{R}^S$ is uncountable in S. Under certain conditions, which we are calling forcing conditions, there is a state of the situation in which any subject who inhabits that state (Badiou 2005) has no discernible access to the real. This is because the subject only has access to countably many multiples (by definition the ordinal count of a situation is less than ω) and only discerns the finite countable reals. Those reals are countably equivalent to the finite ordinals.

We will call $\mathbb{R}^S$ the set of countable reals that are accessible (or discernible) to the subject who can validly deduce that the sentence of the form "x is countable" is either true or false. The "ontologist" (Badiou mysteriously draws a distinction between the ontologist and the abstract subject of a situation) knows this sentence as " $\forall f\,(f : x \rightarrow \omega)$ where f is a bijection." In other words, the ontologist has access to some functional operation that the subject does not. The idea that something of an infinite situation can be known by a functional dexterity that is not limited to axiomatic constructibility, lies at the heart of genericity. For the moment, it will suffice to assume that the subject can perceive only the finite discernible parts of the situation $\mathbb{R}^S$, that are located in the intersection $\mathbb{R}^S \cap \mathbb{R}$.

This suggests that the subject qua ontologist of the situation has some "cognitive" faculty f that can perceive a bijection between x and ω. For a subject inhabiting S, the functional range of f is, of course, restricted to what the subject knows of S. The subject will never fully perceive the functional correspondence between a finite x and the infinite ω, because the "cognitive" bijection f ranges well beyond the consistent limits of the situation. Any representation of f will ultimately fail in S because a some point the subject will misperceive the cardinality of ω. Such a measure will always lie beyond the discernible finite count of the situation.

If the situation S is also a part of a larger situation S', i.e. $S \subseteq S'$, and the subject of S can discern that the measure of two states s and t are equal (we are talking about two cardinal states in a situation, not abstract model theoretic entities like $\mathbb{R}$ and ω_α) and $S \vDash |s| = |t|$, then also $S' \vDash |s| = |t|$ because S has every bijection that S' has. This is because $|S| \leq |S'|$ when $S \subseteq S'$. The evental collapsing of larger situations into smaller ones can, under the conditions of forcing, be effectively achieved provided care is taken to identify (name) the parts of the extended situation which are indiscernible in the non-extended situation.

Generic Situations

Consider a situation S to be both countable and transitive and let "♂" denote some set that is not part of the situation, i.e. ♂ $\nsubseteq S$. Construct the smallest extension S [♂] of S, which contains all the elements of both S and ♂. To gain some meta-ontological control of the generic extension of the situation we will let some $\mathbb{P} \in S$ be a separative atomless partial order and let B be a Boolean algebra. Consider the set ♂ to be a "generic" ultrafilter over B (recall, Boolean ultrafilters are Boolean functions which always return the value **1**).

Ontological control can be extended over a situation and its generic extension only when certain conditions are in place. These conditions have to exist if the ontological integrity of a situation is generically extendible to infinite domains. In this way certain indiscernible infinite states can be meaningful in the original situation. This control involves the idea of a partial ordering of S. The sort of partial order that needs to be considered is be both separative and atomless. The idea is that one state can be extended from another state so that more may understood from the extension that can be understood before. Given a situation $S \subseteq M$ in which M is a generalized model of ZFC, then if M is consistent with the axioms of ontology, so is S consistent. The partial order $\mathbb{P}$

will be separative and atomless in any model that contains it. If the forcing conditions which make up $\mathbb{P}$ and B corresponds to a complete Boolean algebra, then ♂ will be a generic ultrafilter over B. Since ♂ is an ultrafilter such that for every dense part D of S, and if D is also a dense subset of $B \setminus \{\mathbf{0}\}$, then ♂ $\cap D \neq \varnothing$. Since S is countable, we know ♂ exists because there are countably many subsets of B and ♂ is a countable subset of B.

These dense subsets can be enumerated $D_0, D_1, D_2, \ldots$ etc. Since each set is dense, then there will be some state $s_0 \in D_0$, and $s_1 \in D_1$ such that $s_1 \leq s_0$, then $s_2 \in D_2$ because $s_2 \leq s_1$, and so on. [18] Because each D is dense, and we can isolate a state contained in each, we can then correspondingly index it to the ordinals in such a way as that each pair of ordered states s_i and s_j form an extension relation $s_j \leq s_i$.

The sequence $s_0, s_1, s_2, \ldots, s_i, s_j, \ldots$, of dense states can then be fixed by the countable axiom of choice.[19] We can now define ♂ as the set

$$♂ \ = \{x \in B : \exists i \, (x \leq s_i)\}$$

This means that there is always some dense state which is the extension of any set x in our Boolean correlate of states. Because ♂ contains each state $s_i \in D_i$, which are indexed to the dense parts of the situation, then it follows that ♂ intersects every dense subset D_i, of B. Because each state $s_i \in D_i$, is a state in the (Boolean) partial order $\mathbb{P}$, then it is clear that ♂ closes upwards under the order $\leq$. ♂ is indeed a filter, because by definition any elements $x, y \in$ ♂ are such that the extensions $x \geq s_i$ and $y \geq s_i$ hold for the ordinal

[18] Note the reverse order under $\leq$.

[19] Axiom of Choice: given any collection of mutually disjoint nonempty sets, it is possible to assemble a new set – a choice set – containing exactly one element from each member of the collection. “Nevertheless, this seemingly innocuous principle has far-reaching mathematical consequences … and has come to figure prominently in discussions on the foundations of mathematics.” (*Stanford Encyclopedia of Philosophy*) Note also that the axiom of choice is independent of the other axioms of ZF, but its use does not effect the outcome of this proof.

indices i and j. As $x \sqcap y \geq s_i$ (i.e. the greatest lower bound value of x and y extends the state s_i) then $x \sqcap y \in$ ♂, so is a filter.

For any $p \in \mathbb{P}$, the dense subset $D_p = \{x : x \leq p \text{ or } x \leq \neg p\}$ is also a dense subset of B. Therefore $D_p \in S$ and ♂ $\cap\, D_p$ is non-empty. So either p or $\neg p$ is in ♂, meaning ♂ is indeed an ultrafilter.

Ontological Truth

The generic extension of a situation S[♂] can have a number of ontological properties, which can determine how a site and an event need to be considered when their corresponding situation is generically extended to infinite domain of the real. However, given the idea that an ontological situation is a partial order of a certain kind, then the structure of that partial order needs to be understood. If a partial order $\mathbb{P}$ catalyzes the relational extension of one state to another, care needs to be taken to understand how $\mathbb{P}$ behaves when the extensional relation $\leq$ ranges over transfinite states.

Let κ be a cardinal set such that

$$S \vDash (|\kappa| = \omega_\alpha)$$

Let the B-transcendental (or partial order) $\mathbb{P}$ be a set of finite partial functions from $\kappa \times \omega$ to $\{\mathbf{0}, \mathbf{1}\}$. That is, each element $s \in \mathbb{P}$ specifies finitely many values for the product array $\kappa \times \omega$ (from the possibly finite κ to the infinite ω) of zeros and ones. We will later consider these zeros and ones to be the truth value coefficients, “false” and “true” respectively.

If s and t are two elements of $\mathbb{P}$, then let $s \leq t$ if and only if $t \subseteq s$ and s is a function (functions can be particular types of states) that extends the function t. Notice how this ordering reverses the direction implied by the normal "greater than or equal to" interpretation of the sign "$\leq$".

The idea that states lower down an order give more information than states further up the order, is crucial to our understanding of the principles of forcing. [20]

The reverse ordering of $s \leq t$ is atomless under this configuration, because any finite partial function s, t can be extended by adding one or more zeros or ones to the $\kappa \times \omega$ array. The partial order $\mathbb{P}$ is also separative under this treatment because the compatibility relation $s \perp t$ holds if and only if there is some point in the array where one function assigns a zero and the other function assigns a one. This operation has a pertinent intertextual link to Badiou's notion that a subject is "a finite instance of a truth." (Badiou 2005, 523)

We have seen that ♂ is a Boolean ultrafilter with certain properties. If $s, t \in \mathbb{P} \cap$ ♂, then the greatest lower bound of s and t is also in ♂. Further, if $\mathbb{P}$ is dense in B, it follows that state s is not compatible with the state t.

Because any two partial functions, that are included in the intersection of the partial order $\mathbb{P}$ and the ultrafilter ♂ are compatible, their union specifies some unique function from $\kappa \times \omega$ to $\{\mathbf{0}, \mathbf{1}\}$. By definition, the partial order $\mathbb{P}$ is also a dense part of the Boolean algebra B, which we have already associated with the situation S. Because the intersection of $\mathbb{P}$ and ♂ also intersects every dense part D of $\mathbb{P}$, then for any $s \in \kappa, n \in \omega$ and function $p \in \mathbb{P}$

[20] Note that the cotranscendental orders, which will be introduced in our discussion of the *sinthôme*, are reversals of a Badiouian transcendental order of a world. The notion of a cotranscendental ordering of a post-evental world agrees with the idea that the sign "$\leq$" may indeed imply a reverse ordering in which a greater density of information is gained from a *predecessor* order that might otherwise be gained from a *successor* order. As we shall see, a coinductive predecessor order pushes meaning into the dense subregions of language which lay closer to the place of the lower bound void that Maria Wyeth names 'nothing.'

$$D_{s,n} = \{p : p\ (s, n)\}$$

is also a dense part of $\mathbb{P}$. As we have seen, any finite partial function in $\mathbb{P}$ can always be inductively extended to another partial function with which it is compatible. Therefore $\mathbb{P} \cap ♂$ intersects every dense part of $\mathbb{P}$ because all the finite partial functions become total [21] in $\mathbb{P} \cap ♂$. If $f_{♂}$ is such a function, then

$$f_{♂} = \kappa \times \omega \rightarrow \{\mathbf{0}, \mathbf{1}\}.$$

Defining such particular functions generally, we see that $♂_s = \kappa \times \omega \rightarrow \{\mathbf{0}, \mathbf{1}\}$ is derived from letting $♂_s(n) = f_{♂}(s, n)$. If $s, t \in \kappa$, then

$$D_{s,n} = \{\ p : \exists n\ (\ p(s, n) = p(t, n))\}$$

is also dense, because any finite partial function can be extended by finding some n for which neither $p(s, n)$ nor $p(t, n)$ is defined and letting one value be $\mathbf{0}$ and the other value $\mathbf{1}$. Since $\mathbb{P} \cap ♂$ intersects both $p(s, n)$ and $p(t, n)$, $♂_s$ and $♂_t$ are different functions from ω to $\{\mathbf{0}, \mathbf{1}\}$. Obviously $S[♂]$ contains $♂$, so the extended situation can be used to define $♂_s$ and $♂_t$. Each $♂_s$ and $♂_t$ corresponds to a part of ω, so $\mathcal{P}(\omega)$ must be at least as large as κ. Because any situation S is a proper part of its generic extension, then

$$S[♂] \vDash (|\mathbb{R}| \geq |\kappa|)$$

when S is a singularity.

This means that with the generic extension of a situation, the infinite measure of $\mathbb{R}$ can extend $|\kappa|$ into transfinite space, in such a way that something about the truth or falsity of the extension bacomes a truth value of the extension. Since there is a bijection between κ and $\omega_\alpha{}^S$ there is also a bijection in $S[♂]$ such that

[21] See fn. 10 in Chapter 2.

$S[\sigma] \vDash (|\kappa| \geq |\omega_\alpha{}^S|)$

This allows us to determine that a transfinite indiscernible $\omega_\alpha{}^S$, which can be named in the situation *S,* is exactly the same as that which is "discerned" in its extended correlate $S[\sigma]$, so we can see immediately that $\omega_\alpha{}^S = \omega_\alpha{}^{S[\sigma]}$. This implies that the evental consequences of there being no ontological difference between a transfinite indiscernible *named* in a situation is *deduced* from its generic extension, means that the representational difference "between being (in situation)" and its "unassignable errancy" occurs when there is no ontological difference to discern.

Ontological Names

Any consistent ontological situation *S* is either a proper part of a countable transitive model of a complete set theory, *M*, or it is equivalent to the *M*, i.e. $S \subseteq M$. Because any situation *S* is the is consistent with the finite models of Zermelo-Fraenkel set theory, then any logical sentence, which is validly "expressible" in *M* is also expressible in *S* because $S \subseteq M$ is also consistent with the axioms of ZF. However, in the moment of an event when *S* admits itself as an element and thereby becomes the singularity $S = \{S\}$, *S* becomes too large for *M* to handle. The smallest state measure $|\kappa|$ of any part of *S* is either equal to, or greater than ω. If *S* is finitely countable, then any measure $|\kappa|$ of its parts, is less than ω_0. If $|\kappa|$ is greater than ω_0, then the transfinite value of $|\kappa|$ cannot be determined by any consistent model of ZF set theory. However, there are generalized proceedures to make $|\kappa| = \omega_0$ meaningful in *M*. If this is the case we can draw a contextual difference between, on the one hand $\omega_0{}^S$ and $\omega_\alpha{}^{S[\sigma]}$ and on the other hand, $\omega_0{}^M$ and $\omega_\alpha{}^{M[\sigma]}$ if it is clear that we are referring to a particular situation *S* or a general model *M*.

Let ♂ denote a generic ultra filter over a Boolean algebra B. Although ♂ is specified in terms that are meaningful in S, much of its generic extension $S[♂]$ is also meaningful in S. In particular, a subject inhabiting S – which by inclusion also inhabits M – will have a name for everything in $S[♂]$ including indiscernible states κ. However, the subject of S has no way of determining anything in ♂ other than being able to determine whether the truth value of a name is either **1** or **0** (i.e. specify whether the truth value is 'true' or 'false'). If the function naming an indiscernible returns a value **1** (one), we shall conclude that "the name sticks." If the same function returns a value **0** (zero), we shall conversely conclude "the name does not stick." [22]

An ultrafilter in B specifies only the **1** values of B (conversely, a prime ideal specifies only the **0** values of B). Let $[\![\phi]\!] = p \in \mathbb{P}$ be a determinable "truth" function assignable by the subject of S – remember, S is partially ordered under the B-transcendental $\mathbb{P}$ – then it follows that $S[♂] \vDash \phi$ if and only if $p \in ♂$. There may be many ultrafilters over $\mathbb{P}$, and each may make a number of different models of $S[♂]$. This means that from the perspective of the subject, all generic extensions are equally generic, because a subject has no way of distinguishing one generic form from another.

Being able to decide the truth-values of names (logical sentences may be names) allows a subject enough control to be able to decide the truth of transfinite propositions such as

$$\omega_\alpha{}^S = \omega_\alpha{}^{S[♂]}.$$

A name $n \in S$ shall be defined as a set of ordered pairs $\langle m, p \rangle$ such that m is a name and p is an element in $\mathbb{P}$. The axiom of foundation ensures that any chain

$$s_0 \ni s_1 \ni s_2 \ni s_3 \ldots$$

[22] As we shall see, an integral coalgebraic function of a mutant automaton is a name and a choice to go-on. If a name "sticks" then the dynamic flow of the going-on will terminate. In other words, a name can halt the protean flow of the *sinthôme*.

eventually terminates in finitely many steps. Because ontology is consistent with the axioms of ZF, our ontology is well-founded. Therefore, under the strict ontological conditions that determine a situation, downward chains of the sort just described must terminate. We shall see that if a post-evental situation admits non-well-founded sets, such as singularities, the situation becomes ill-founded and is only describable in terms of anti-foundation axioms which violate the axioms of consistent ontology. When this is the case, the formal schematization of a *mutant coinductive naming function over dynamic states*, is plausible. This notion shall inform the definition of the post-evental trace *sinthôme*. In the meantime, we shall consider that any ontological situation S and its generic extension $S[♂]$ are both well-founded.

Given an ultrafilter ♂ over B a name n can be ontologically interpreted as the set

$$n^{♂} = \{ m^{♂} : \langle m, p \rangle \in n \text{ and } p \in ♂ \}$$

which fixes a name to an element p in the generic object ♂ of our B-transcendental. Once ♂ has been 'nominally' fixed, then $S[♂]$ will be the set

$$\{ n^{♂} : n \in S \text{ is a name} \}$$

because a situation is part of its generic extension, i.e. $S \subseteq S[♂]$, then the generic ultrafilter ♂ is an element of the same, i.e. $♂ \in S[♂]$.

To demonstrate that $S \subseteq S[♂]$ and $♂ \in S[♂]$ (cf. (Badiou 2005, 384)) and for any ultrafilter ♂ of B, we can associate a *state* $\check{z}^{♂} = z$ with every element $\check{z} \in S$.

We already know that **1** is the greatest element of the Boolean algebra B. If we consider the name $\check{z}$ to be the set

$$\{ \langle \check{u}, \mathbf{1} \rangle : u \in z \}$$

then if a state r is an element of the state s, then the name $\check{r}$ is always associated to **1** by any relation g such that $\langle \check{r}, \mathbf{1}\rangle \in g$ for any state $r \in s$.

For example, we can develope this schema inductively to see that the subjective *name of the void*, denoted "$\breve{\varnothing}$" (a subject of the situation always does the naming) can be equated to the ontological sign of the void, "$\varnothing$". Given the nomination schemata above, we see that $\breve{\varnothing} = \varnothing$, and following the set-theoretical convention we let $0 = \varnothing$ and the natural number $1 = \{\varnothing\}$, then by our ontological naming convention $\breve{1} = \{\langle \breve{\varnothing}, 1\rangle\} = \{\langle \varnothing, 1\rangle\}$. Extending this inductively, if we let $2 = \{\varnothing, 1\}$, then $\breve{2} = \{\langle \breve{\varnothing}, 1\rangle, \langle \breve{1}, 1\rangle\} = \{\langle \varnothing, 1\rangle, (\{\langle \varnothing, 1\rangle\}, 1)\}, \ldots$, and so on. By simple induction it follows that $\check{z}^{\,♂} = z$ is in ♂ and the names "void," "the one" and "the two" etc., are correspondingly in ♂.

Because **1** is the greatest value in B and $\mathbf{1} \in$ ♂ for any ultrafilter ♂, it follows that

$$\check{z}^{♂} = \{\, \check{u}^{♂} : u \in z\}$$

By our induction assumption $\check{u}^{♂} = u$, so

$$\check{z}^{♂} = \{u : u \in z\} = z$$

Thus, each state of a situation S has a name, which is also in its generic extension S[♂].

To show ♂ $\in S$[♂], consider the name $\dot{Z} = \{\langle \check{p}, p\rangle : p \in \mathbb{P}\}$. Obviously

$$\dot{Z}^{♂} = \{\check{p}^{\,♂} : \langle \check{p}, p\rangle \in \dot{Z} \text{ and } p \in ♂\}$$

However, $\check{p}^{\,♂} = p$ and $\langle \check{p}, p\rangle \in \check{T}$ if and only if $p \in \mathbb{P}$, so

$$\dot{Z}^{♂} = \{p : p \in \mathbb{P} \text{ and } \mathbb{P} \in ♂\} = \mathbb{P} \cap ♂.$$

Constructing ♂ from $\mathbb{P} \cap$ ♂, it follows that ♂ $\in S$[♂].

Note also that if $\check{\omega} = \omega$ is a name in a situation S, (by the axiom of infinity ω is consistent with S), then infinity is also in the generic extension, i.e. $\omega \in S[♂]$.

Not only can names be used to 'nominate' any state in $S[♂]$, names can be used as the sole elements of a "forcing language." Names, the primitive syntactical relations "∈" and "=" and the normal first-order logical operators and quantifiers, can be used to make ontological statements about $S[♂]$ that are completely meaningful in S.

The first-order sentence "$\forall x\ (\exists y\ (y \in x) \Rightarrow \exists y(y \in x \wedge \neg \exists z\ (z \in x \wedge z \in y)))$" (Kunen 1980, 100), which is the axiom of foundation [23], contains no names. So, if ϕ is a sentence of ontology, which is derived solely from the axioms of set theory, then we can assume that

$$S[♂] \vDash \phi.$$

However, if ϕ is a sentence of a forcing language, ϕ needs to be also interpretable under its generic extensor ♂. If the names $\dot{b}, \dot{c}, \dot{d}$ are in S, then the sentence ϕ is defined

$$\phi = \text{“} \exists x(\dot{b} \in x \text{ and } \forall y(\dot{c} \in y \text{ and } (x \in y \text{ or } y \in \dot{d})))\text{”}$$

is also a sentence in the forcing language of the subject. We can now say that the sentence ϕ is meaningful in the generic extension of the situation, i.e. $S[♂] \vDash \phi$, when for $x, y \in S[♂]$

$$\exists x\ (\dot{b}^{♂} \in x \text{ and } \forall y(\dot{c}^{♂} \in y \Rightarrow (x \in y \text{ or } y \in \dot{d}^{♂})$$

We can also say that a condition $p \in \mathbb{P}$ "forces" ϕ – denoted $p \Vdash \phi$ – if $S[♂] \vDash \phi$ for every generic ultrafilter ♂ such that $p \in$ ♂. To know that a forcing condition is an element

[23] Introduced in 1925 by Hungarian born Hungarian/American mathematician John von Neumann (1903-1957). The axiom of foundation is independent of the other axioms of Zermelo-Fraenkel.

of the generic extension of a situation is all a subject needs to evaluate the sentence ϕ to be true or not true in any extended situation $S[♂]$. For every sentence f there is some $[\![\phi]\!] \in B$ such that $p \in \phi$ if and only if $p \leq [\![\phi]\!]$ in such a way that the forcing statement $p \Vdash \phi$ corresponds to the operations of our Boolean valued situation $S \subseteq S[♂]$ under a B-transcendental order. Because the forcing condition p is extended by the determinate truth value $[\![\phi]\!]$ of the sentence ϕ, this is enough to give the subject of S complete control over $S[♂]$. This association, although ontologically difficult to prove (for proof see (Kunen 1980, 192-201)) still justifies calling $[\![\phi]\!]$ the "truth-value" of ϕ. This lays the ground for the coalgebraic treatment of names and the truth-value returns of the dynamic coalgebraic trace of a pseudo-dialectic turn, the *sinthôme*.

Every sentence ϕ is given by $S[♂]$ if and only if there is some forcing condition $p \in ♂$ and $p \Vdash \phi$, i.e. $S[♂] \vDash \phi$ iff $p \in ♂$ and $p \Vdash \phi$. We can also show that the sentence $[\![$ "It is not the case that $\phi]\!]$

When the consistent ontology of an event *collapses*, the "errancy of excess" marked by the impossible cardinal representation $|\kappa|$, such that $\omega < |\kappa| < 2^{\omega}, \ldots, \leq |\mathbb{R}|$, itself represents the indiscernible impossibility that haunts the hypothesis that there exists an inductive ordinal continuum that extends beyond the greatest countable cardinal state of representation ω.

Forcing the Real

As we have seen, $S[♂] \vDash (|\mathbb{R}| \geq |\omega_\alpha{}^S|)$ but we need to show that $S[♂] \nvDash$ CH. To do this, all that needs to be shown is

$$\omega_\alpha{}^S = \omega_\alpha{}^{S[♂]}.$$

We know that $\omega_\alpha{}^S \leq \omega_\alpha{}^{S[♂]}$ because the generically extended situation $S[♂]$ can have only more bijections than S. However, if there must be some β such that $S[♂]$ has a bijection $\omega_\beta{}^S$ and $\omega_\beta{}^{S[♂]}$, which by the principles of forcing, we can show that such bijections are impossible. Let κ and κ^+ be states of representation with cardinalities $\omega_\beta{}^S$ and $\omega_\beta{}^{S[♂]}$. Because $S[♂]$ extends the situation S, then $\omega_\beta{}^S$ and $\omega_\beta{}^{S[♂]}$ will also be in $S[♂]$. The union of ω_β many discernible (countable) states has the cardinal measure $\omega_\beta{}^S$, which is discernible in the situation S. The subject inhabiting S can ontologically understand this through the any ZF-consistent language of S. However, there is no consistent way for the subject to deduce any transfinite state of representation $\omega_{\beta+1}{}^S$. The power of representation of any transfinite state with the count $\omega_{\beta+1}{}^S$, therefore remains errant and in excess of anything the subject can discern. In this way, the measureless quantitive difference of power between the state, which is the situation, and the state of representation of the situation (Badiou 2005, 81-92) can be named. With the nomination comes collapse. When the name sticks, the errancy of excess evaporates.

As we have seen, there is no way for a subject of S to ontologically determine (discern) the cardinal count of states $\kappa \in S$ and $\kappa^+ \in S[♂]$ with the cardinal counts $\omega_\beta{}^S$ and $\omega_{\beta+1}{}^S$ respectively. however, by the axiom of choice it is possible to know the union of $\omega_\beta{}^S$ countable states, but not the union of $\omega_{\beta+1}{}^S$ states. If a subject, in the moment of an event, is able to *subjectivize* a "finite instance of a truth" (Badiou 2005, 523), then ontologically speaking, there must be a function $f \in S$ that assigns to each state of κ, a countable subset of κ^+. Naturally, the union of κ and κ^+ will exclude some states of κ^+.

Assume that there is some such function $f \in S$ whose domain κ is discernible to the subject but whose codomain κ^+ is not. Because there is a name for every object (functions are objects) in $S[♂]$, let the function $f \in S$ have a name $\dot{f}$ such that every state of $\breve{\kappa}$ maps to a state in $\breve{\kappa}^+$ and vice versa. Thus $S[♂]$ gives the true sentence

$$\text{“}\dot{f}: \breve{\kappa} \rightarrow \breve{\kappa}^{+} \text{ is a bijection ”}$$ [24]

By the principles of forcing, some condition $p \in \mathbb{P}$ can be constructed so that p forces the statement

$$p \Vdash \text{“}\dot{f}: \breve{\kappa} \rightarrow \breve{\kappa}^{+} \text{ is a bijection ”}$$

Define $f_p \in S$ to be a function from κ to the subsets of κ^+, then for any $z \in \kappa^+$ there is some $z \in f_p(u)$ if and only if there is some $p_{u,z} \leq p$ such that the condition $p_{u,z}$ forces the veracity of the sentence

$$p_{u,z} \Vdash \text{“}\dot{f}(\check{u})) = \check{z}\text{”}$$

to be true.

Because $\mathbb{P}$ is dense in B, we can assume that $p_{u,z}$ is an element of $\mathbb{P} \subseteq B$. But $p_{u,z}$ must be compatible with $p_{u,z'}$ (in symbols $p_{u,z} \perp p_{u,z'}$) if $z' \neq z$ since both conditions force f to be a bijection. More importantly, these conditions force $\dot{f}$ to take different values at the same state in $S[♂]$. In other words, the function f is undecidable at p.

Any collection of incompatible elements of $\mathbb{P}$ must also be discernible (countable) in S. Thus each of the sets $f_p(u)$ must also be discernible in S, so their union cannot be the whole of κ^+. Because $f \in S[♂]$ is a bijection between κ and κ^+, then for every $z \in \kappa^+$ there is some $u \in \kappa$ and $p_{u,z} \in ♂$ which is extended by p and $p_{u,z} \Vdash$ “$\dot{f}(\check{u})) = \check{z}$”, thus every state of κ^+ is in some function $f_p(u)$. This is a contradiction, so *via negativa*, the assumption that there was some f e $S[♂]$ which is a bijection between κ and κ^+ is false, concluding that S[♂] has the same cardinalities as S, so $S[♂] \vDash (|\mathbb{R}| \geq |\kappa|)$ when S is the singularity $S = \{S\}$. Thus a subject “saves the singular” from the errancy of excess. Anything of the real lying beyond the natural edge of reason

---

[24] Note: the name $\dot{f}$, with a diacritical “·” dot, denotes a name for a function over $S[♂]$, while the names $\breve{\kappa}$ and $\breve{\kappa}^{+}$ with a diacritical “ ˘ ” breve, denote names in S.

Because the generic extension of the situation has the same cardinal states of representation as the original situation, the errancy of excess of all states of representation of S collapses. A subject saves the singular from the errancy of excess because a name rings true.

Concluding Comment

In Chapter One the elaborate formal detailing of forcing Boolean-valued situations has two efficacies. First, we saw how a Boolean B-transcendental ordering of a situation can produce an ontological mechanism for showing how an ontological 'name' can determine the truth of indiscernible representations. Although I introduced the notion of reverse ordering of a B-transcendental by turning the intuitive meaning of the relation $\leq$, this procedure is arbitrary. Many authors describe modern forcing techniques the normal ordering of the $\leq$ relation. Reversing the B-transcendental ordering of a situation, gives ground to the idea of a pseudo-dialectic turn that reverse the transcendental order of a world. The reverse ordering of a Badiouian transcendental I call a cotranscendental order. Cotranscendentals order worlds that are ontologically non-well-founded.

Second, the proof that naming ontological indiscernibles can be meaningful in generically extended situations with the normal ordering of the $\leq$ relation. Names, whose referents are indiscernible can be meaningfully understood as the pure affect of the process of nomination. A name whose referent is the non-well-founded effect of the occurrence of a singularity, give *prima facie* grounds to postulate the actuality of non-well-founded situations, such as the imaginary ill-formed worlds of the *sinthôme*.

Chapter TWO

Ill-founded Worlds and Mutant Traces

Extraordinary Situations

In its simplest ontological form, a situation is a set. The iterative concept of set theory entails the notion that all sets must be arranged in levels with the elements of a set placed at a lower level than the set itself. This means that the string of sets $E \in E^{*} \in E^{**}\ldots$ is well-founded. For the iterative, only well-founded sets exist and the foundation axiom of Zermelo-Fraenkel set theory is only true when interpreted in the iterative universe of pure sets. In the introduction I discussed the notion of non-well-founded sets and ill-founded worlds. Mirimanoff describes non-well founded sets as the non-terminating descending order

$$\ldots E^{**} \in E^{*} \in E$$

Mirimanoff asserts that a set is ordinary (by implication a situation is also ordinary) when it only entails finite terminating descents. A set "… is *extraordinary* when among its descents there are *some* which are infinite." (Mirimanoff 1917)[1]

Extraordinary sets can never be formed by the iterative constructor principles under which only well-founded sets may exist. Infinite sets are extraordinary because they are non-well-founded. For Badiou, the foundation axiom, as well as the other axioms of

[1] Quoted in (Aczel 1988, vii).

Zermelo-Fraenkel set theory, are true only when they are interpreted in the iterative inductive universe of sets. Badiou's entire ontological theory of the event rests on the assumption that the set-theoretical world is well-founded. Yet, the disruptive moment of an event renders a site to be radically non-well-founded. This means that situations that are sites are non-well-founded. As we shall see, worlds that admit non-well-founded ontologies are ill-founded. There is the temptation to develop an ontology of sites that include an anti-foundation axiom with those of Zermelo and Fraenkel but this cannot be achieved without contradiction unless the axiom of foundation is replaced by its anti-foundation correlate. The ontology of an evental site presupposes such a contradiction. Therefore, any site is not constructible in the sense that its ontological structure can be consistently derived from the axioms of Zermelo-Fraenkel set theory. To do so would admit to the existence of singularities, i.e., the non-well-founded reflexive sets such as the singularity $A = \{A\}$, which violate the axiom of foundation. This is exactly what happens in the moment of an event.

Under Badiou's theory of the event, the occurrence of a singularity, such as, is momentary. Of course the proposition $A = \{A\}$ violates the axiom of foundation. This gives the evental site its radical disruptive character. Badiou's notion of the proper inexistent $\varnothing_A = \{A\}$ of a presented multiple A is indeed a non-well-founded set. Inder a badiouian reading of the event, he difference between the singularities $A = \{A\}$ and $\varnothing_A = \{A\}$ is that the former is purely ontological while the second entails the existential notion of appearance. The point I am making is that Badiou's theory of an event admits non-well-founded sets, but only at the topological location of a site and in the moment of an event. This is because the formation and constructor principles that are crucial to the iterative notion of sets, remain in place despite the momentary disruption of an event.

The subject on the edge of the void – if this formulation of the subject is still plausible after *Logics of Worlds* – will after the event, return to a world order whose ontological constitution remains well-founded. In other words the ontological composition of a world remains unaltered after and event. However, according to Badiou, real change occurs but only on the subjective level of retroactive representation.

The notion of retroactive subjectivity is of significant concern. The faithful post-evental subject must continually work against the caustic flow of historicity that order any world. This suggests that following an event, "faithful" subjectivity may immediately find itself under the reactionary threat of historical occlusion. The representational power of historicity will move to occlude real change. Under such pressure, the notion of faithful subjectivity is soon reduced to a condition of maintenance nostalgia for the evental moment now passed. Any evental trace that maintains the spirit and force of the event may soon be absorbed by the same onward dialectic flow from which the evental subject had momentarily escaped.

If a subject is able to faithfully bear the subjective formalism that enables that subject to retroactively incorporate itself into the spirit of an evental present, it will be because the world in which she finds herself has significantly altered. If this is the case, then there are at least two possibilities that need to consider.

(ix) The world has so significantly altered that new ontologies have to be considered and new post-evental models have to be recognized. This would entail altering the classical models of set theory in which the axiom of foundation needs to be radically reconsidered.

- The evental trace must follows an affective formalism that includes the notion of bisimilarity. Bisimilarity can be seen as an intensional (or affective) equivalence relation. This is contrasted with ontological set equality, which is extensional (or effective). Bisimilarity allows the notion of the evental trace to be construed as the affective consequences of an event which does not follow the constructible logic of the situation. If this the case, then under bisimilarity, unknown states and transition may be subjectively recognized and an affective formalism may be set in place.

One way to address this challenge is to construct a combined solution that addresses both points. Following Mirimanoff's impetus we can develop alternative post-evental models of set theory that accommodate *extraordinary* multiples, non-well-founded sets and anti-foundation axioms. This is the direction I will take.

In the literature there are a number of set theories that admit anti-foundation axioms, including principally those of Peter Aczel, Maurice Boffa, Ulrich Felgner and Dana Scott (Aczel 1988; Boffa 1969; Felgner 1969, 1971; Scott 1960). Scott's approach is to abandon the foundation axiom of Zermelo-Fraenkel set theory altogether but strengthen the axiom of extensionality to accommodate the idea of bisimilarity. Given a fixed regular bisimulation, denoted "$\approx$", a system M is $\approx$-EXTENSIONAL (or intentional) and if two strings (traces) M_a and M_b are bisimilar, then by extension, there is one injective system map $\pi : M_0 \rightarrow M$ (Aczel 1988, 41-42) such that $M_a \approx M_b$ implies that $a = b$. Aczel:

> We will develop the ontological model of set theory that excludes the foundation axiom but includes an anti-foundation axiom in its place. This model can be denoted "$ZFC^- + AFA^{\approx}$". The model $ZFC^- + AFA^{\approx}$ admits the standard axioms of ZFC but excludes the axiom of foundation and replaces it with an anti-foundation axiom AFA.

Following Aczel, we will denote the models of Zermelo-Fraenkel set theory that do not include the axiom of foundation "ZFC$^-$".

The coinductive trace of an event (or a turn) is grounded in the bisimilarity relations which do not necessarily obey the classical set-theoretical laws of ontological. This mathematical orientation, together with a specific readings of Samuel Beckett's *The Unnamable* and Joan Didion's *Play It As It Lays*, inform intertextual impetus of this chapter.

Bisimulation

Consider the ordered pair $\langle x, y \rangle$ to be under the identity relation $\mathbf{Id}(x, y)$ (Badiou 2009a, 243). That is, $\langle x, y \rangle \in \mathbf{Id}$. If x and y are state objects (or elements), then by the identity relationship **Id** they are existentially equivalent if their transcendental degree of identity is maximal and the ontological base logic of their world is well-founded. Under an existential regime, the degree of identity between x and y is maximal if $x = y$. If on the other hand, x and y are (ontological) multiples in a state space (or situation) S, then by the axiom of extensionality $x = y$ implies that the predicate values of x and y are the same. In this interpretation, the variables x and y no longer denote the "degrees of identity" of the transcendental indexing of some multiple in a world but denote two states of the situation S, which happen to be ontologically equivalent. Substituting the multiple variables x and y for the states s_1 and s_2. Consider s_1 and s_2 to be equal by letting the **Id** function operate on the degree of equality between state pairs in the post-singular state space S. By "post-singular" situation I mean a state space that is a situation of presented multiples in which an ontological singularity has occurred. As we shall see, such singularities may may appear to be either silent or loud, or in Badiou's nomenclature, weal or strong respectively. As we have seen, strong singularities form sites with maximal transcendental degree of existence (or appearance), while weak singularities do form sites but they exhibit a minimal transcendental degree of appearance. This polarity reverses under a cotranscendental ordering of a world.

Bisimulations can be defined as protean relations which are the duals of the common algebraic relation of partially ordered pairs.

In this formal restructuring of a post-evental situation, many states in the situation are unknown and are in flux. Such states can be linguistically charged and "subjectivized" by associating a cypher (name) with the post-singular trace that inevitably survives the radical ontological disruption brought on by the event or the silent protean shift precipitated by the pseudo-dialectic turn. This accumulatively transforms the meaning of any material body into an abstract subject-body.

An evental trace is not differentially determined by its relationship to other traces. Two traces are equivalent if their states are equivalent. The equivalence of states occurs when the relationship between them is one of bisimilarity. There are a number of formal processes needed to define bisimilarity and the equivalence and associativity of states. These in turn are needed to schematically define the cotranscendental ordering of a world in which an event or a turn has just occurred. A distinction between the existential status of a quiet singularity (with a minimal coefficient of appearance) and that of a loud singularity (with a maximal coefficient of appearance) needs to be made. Although these singularities are each associated with a evental turn and event respectively.

In Badiou's estimation, a 'singular' situation forms an evental site if its intensity of appearance is maximal. In the moment of the event, the absolute "existence" of some multiple *A* in the transcendental field of a situation, is absolute "under its own objective reference (*A*, **Id**)." (Badiou 2009a, 392). Following the momentary condition of the event, "bit by bit" the protocol of the object will have taken place , retroactively appearing as a new objectivation of the site. Notice how the notion of retroactive appearing implies the future anterior evental rupture of past and future time and the retroactive reconstitution of the post-singular situation under the evental trace.

Because bisimilar traces are fundamentally coalgebraic and coinductive, the idea of a retroactive objectivation of a site is replaced by a coinductive "destructor" predecessor structure. A destructor predecessor function characterizes the mutant dynamics that follow the occurrence of a silent singularity. On the other hand the dialectic evolutionary “bit by bit” successor function that is implicit in the ontological structure of a situation – we know this as the *B*-transcendental order of the situation – may quietly “turn” into a predecessor function following the occurrence of a silent singularity. Any materialist notion of dialectic progress, cognitive evolution or sublation is *turned* into the traumatic, contradictory and circular character of unknowable non-well-founded state spaces. The mutable dynamics of such states is implicit in the trace of a silent singularity. This trace I call *sinthôme*.

The relationship between two states is bisimilar if their affects are equal. The logic of bisimilar states is intensional and not extensional. This means that the objectivation of a site does not rely on the ontological equivalences. Two equal traces does not need to have equivalent ontological bases. However, their *subjectivation* will be cotranscendentally equal under the relationship of bisimilarity.

* * *

In *Being and Event* the idea of the representation of a element presented in a situation is a mode of “numerary structuration proper” to the state of that situation. In short, this is an application of the power set axiom to the ordinal count of the situation. Every element is represented in a situation as either a subset whose only member is itself or as a member of a subset that includes other elements as members. The “count” of the set of all subsets of a situation is the cardinal measure of the situation. Therefore the count of the situation is the finite cardinal measure of its elements. This count may be transfinite when the situation is a site. Because the cardinal number of a set is always 2^n times greater than its ordinal count n, then the state of representation of a situation is always (cardinally) in excess of the ordinal count of that which is (ordinally) presented. There is a sense in which this excess is errant when the cardinal count of situation falls is one of transfinite inaccessible closure under Cantor's diagonal argument. (Cantor 1874) [2]

The notion of the ontological inaccessibility of certain transfinite cardinal measures also persists in *Logics of Worlds*. Badiou notes: "every world is measured by an inaccessible infinite cardinal." Such a cardinal could be the measure $|\kappa|$ of some object κ, which as we have seen, if situated between two successive transfinite cardinals, i.e. $\omega_\alpha < |\kappa| < \omega_{\alpha+1}$, is not only indiscernible, its existence is also ontologically impossible. The totalization of any world implies "for a being that appears in a world (which is an element of this world in the ontological sense) to be of a magnitude equal to that of the world itself," by the power set principle and the principle of inaccessible closure of a world, the representation of a being in a world is never ontologically nor existentially in excess of the world itself. (Badiou 2009a, 332, 333)

> The cardinality of an element of [a world] remains lesser than that of [the world] itself, it is clearly impossible to construct a world, either from below (dissemination) or from above (totalization), … the magnitude of any world whatever is only measurable by an inaccessible infinite cardinal. This is the principle of inaccessible closure that governs the ontology of worlds. (ibid. 334)

[2] The diagonal argument is a mathematical proof that there are infinite sets, which cannot be put into a bijection (one-to-one correspondence) with the infinite set of natural numbers ℕ. Such sets are uncountable, and by Badiou's account they are indiscernible. The indiscernible lies at the heart of the formal structure of an event as it is described in *Being and Event*. Cf. Meditation Twenty-nine, etc. (Badiou 2005, 295-299).

Any attempt to generically extend a world to ontologically exceed its least upper bound limit must terminate at the unique greatest point of the *B*-transcendental of that world [3] (or an "ontologico-transcendental invariance or stability" as Badiou calls it (Badiou 2009a, 336)). The "atomic logic" (which I persist to call ontological *B*-transcendental logic) of a situation cannot admit singularities. This means that the existential "degrees of existence and localizations" have no relations powerful enough to depose "the real atomic substructure of appearing." (ibid.) On the other hand, an ill-founded world that admits non-well-founded atoms turns at the greatest fixed point of its *B*-transcendental order. The issue of genericity is redundant in an ill-founded world. This is because the cotranscendental order of that world is spawned of a singularity ontologically located at its greatest fixed point. There is no need to transgress the upper-bound edge of well-founded reason because the pseudo-dialectic turn that is instigated by a subject whose affective proximity to the real, marks a radical reversal in the transcendental order of any world occupied by a subject who has turned.

Bisimulation is a relation between states that emerges from the ill-founded worlds forbidden by the laws of "atomic logic." The bisimilarity of states has no need to penetrate the aporetic no-go zone of well-founded ontologies. Nor do bisimilar states radically expand the existential limits of appearance. The situations in which bisimulations hold are logically ill-founded, infinite and circular. Bisimulations characterize relations between non-well-founded states and the traces that emanate from

---

[3] Let the *B*-transcendental T of any world **m** be the partially ordered set $\langle \mathbb{P}, \leq \rangle$. For subsets $S \subseteq T$, the **least upper bound** of S is an element $s \in P$ such that

(i) $t \leq s$ for every $t \in S$

(ii) if for any $u \in \mathbb{P}$ such that $x \leq u$ for every $t \in S$ then $s \leq u$

The least upper bound does not exist if there is no upper bound, or if the set of upper bounds has two or more elements of which none is a least element of that set. It can easily be shown that, if S has a least upper bound, then it is unique. This means that if both s_1 and s_2 are least upper bounds of S then it follows that if $s_1 \leq s_2$, then also $s_2 \leq s_1$. Since the partial order $\leq$ is a binary relation over the set $\mathbb{P}$ it is antisymmetric (by definition any partial order is reflexive, antisymmetric and transitive), then indeed $u_1 = u_2$, showing the least upper bound of S is unique.

sites and pseudo-dialectic turns. Unlike Badiou's staunch extensional "ontologico-transcendental" definition of a relation, our definition of bisimulation has a strong impredicative circular character, because "bisimilarity is itself a bisimulation and therefore part of the union from which it is derived." (Sangiorgi 2009b, 115) Logical circularity and ontological aporia come pre-packed.

We should observe however, that because " … being qua being can be pronounced under the condition of language and the indiscernible" (Badiou 2005, 310) then under the conditions of an event and its trace, an indiscernible may indeed be retroactively discerned and the unnamable may be named. As a subjectivizable body emerges from the place of the evental site and precipitates in an uncertain world order, the old systems of representation that once held absolute representational power over the presented, have momentarily burnt out by the pure white light of a singularity. States of representation invariably reform after any event and thereby set in play their dominant mode of transmission: induction.

The burgeoning body that emerges from the site of a truth made visible, emerges into a world that is structurally (both ontologically and transcendentally) unaltered. However, my idea of a nascent post-evental body and its associative trace, abandons the view that a dialectically reformed world should remain constructible and well-founded. A post-evental world that is structurally congruent with its pre-evental antecedent, is at odds with the idea of a nascent revolutionary body that is continually being reconfigured by the mutating coinductive flows released by the absolute force of a singularity. This means that the force of the event may indeed be significant enough to dialectically sublate the event. For as long as the world into which the revolutionary body is born, remains non-well-founded, the *final* states of the situation that form its evental site may then be cotranscendentally reconfigured. The mutable lines of flight traced by a nascent cotranscendental ordering of the world, emerge from the exact point the transcendental collapsed. A final or *terminal* state space that admits non-well-founded states may then be *determined-in-thought* by the *final semantics* of a terminal coalgebra. The *initial semantics* in which complete partial-order relations between objects and the hierarchy of degrees of existence that form the transcendental of a world as per Badiou, can now be transformed under the cotranscendental reconfiguration of that world.

Ill-Founded Worlds

Clearly the consistent models of Zermelo-Fraenkel set theory and $ZFC^{-} + AFA$ are not equal. As we have seen, the non-well-founded set $A = \{A\}$ cannot be deduced from the axioms of Zermelo-Fraenkel set theory. The axiom of extensionality determines the conditions for the equality of two sets (both sets must have the same elements), so by transfinite induction on the membership relation $\in$, the equality relation between two well-founded sets is uniquely determined. An anti-foundation axiom implies that there is at most one solution to the Quine atom $A = \{A\}$ is the infinite set $\Omega = \{\{\{\ldots\}\}\}$ which we will consider to be the proper class of all sets. For any system of presentation M that is a full interpretation of the language of set theory, each complete system M is a model of $ZFC^{-} + AFA$ (Reiger 1957).

Between 1950 and 1970 a number of set theories were developed in which the axiom of foundation fails. The general method for establishing such theories is encapsulated in Reiger's theorem, which roughly concludes that our theory $ZFC^{-} + AFA$ has a full model that is unique up to isomorphism (Aczel 1988, 37).

Reiger's theorem (and its equivalents) embodies the largest fixed point of some dynamic system M, assuming AFA. We will see that since the power set functor $\mathcal{P}$ does not have an initial algebra, such as our B-transcendental. By Cantor's theorem, $\mathcal{P}$ has no fixed point, (since the cardinal measure of $\mathcal{P}(A)$ is greater than the measure of A, i.e. $|\mathcal{P}(A)| > |A|$) (Adámek 2005, 162). This idea is crucial to our understanding of Badiou's earlier theory of the event. From this we will retrieve the idea of a greatest ontological fixed point – the greatest fixed point of a situation is be the initial object of the coalgebraic trace *sinthôme*.

Remembering that the state of representation of a situation is ontologically determined by the power set axiom, then this errancy of excess of the state is represented by an indiscernible fixed point $\kappa < \omega_0$. Because such a point is of transfinite cardinal magnitude, it is therefore indiscernible to the finitist domains of meaning inhabited by ordinary subjects. In Badiou's early configuration of an event, a subject on-the-edge-of-the-void may somehow force the presentation of the indiscernible "errancy-of-excess" that is occluding unrepresentable lack. Through concealing this lack, any state of representation maintains its power. However, the measureless "difference of power" between the state of a situation and the situation itself marks the congruency of a Cantorian indiscernible transfinite fixed point $|\kappa|$ such that $\omega_\alpha < |\kappa| < \omega_{\alpha+1}$, which is the "discernible" source object of a terminal coalgebra.

Because the greatest fixed point $|\kappa|$ is indiscernible in *ZFC*, but is the greatest knowable point in $ZFC^- + A^{\approx}$, this leads us to see that a turn between the algebras that order normal worlds and the coalgebras of the *sinthôme*, turn at a greatest fixed point which is underivable in *ZFC*.

We let *AFA* be the greatest fixed point of the power set functor *F* of some dynamic system *M* with an unknown state *X*. Because the foundation axiom is ontological disabled by the momentary appearance of a singularity at the moment of the event, it is plausible to think that any post-evental situation may no longer be constructible in a purely inductive algebraic sense. If this is the case then the unknown state *X* is inductively unknowable.

* * *

To see how the idea of non-well-founded situations and and non-well-founded sites plays against Badiou's theory of the event, consider the following propositions (Badiou 2009a, 395).

Proposition: A site is a self-reflexive multiplicity. Because it contains itself as an element, a site “transgresses the law of being.”

Analysis: By transgresses the law of being, a site must ontologically violate the axiom of regularity (foundation) and therefore admit non-well-founded sets. Such sets are sometimes named “hyper-set.” For example, the Quine atom $Q = \{Q\}$ is a hyper-set. As such, a hyper-set exists and is unique under the anti-foundation axiom of Aczel (Aczel 1988). The singularities AA, $A\ \{A\}$ and $\varnothing_A = \{A\}$ are ontological hyper-sets.

Proposition: “Because [a site] carries out a transitory cancelation of the gap between being and being-there, a site is an instantaneous revelation of the void that haunts multiplicities.”

Analysis: The “revelation of the void” is ambiguous. This leads us to infer that there is more than one “form” characterized by “void.” There are at least two such characterizations:

As lack under the sign of the empty set $\varnothing$. Ontology admits only one void $\varnothing$, which is an element of any part of a multiple. In this sense $\varnothing$ may indeed *haunt* any multiple. There is a further sense in which $\varnothing$ is always *present* as a part measure of any state of any multiple that is presented.

Void as excess under the sign of the singularity.

Proposition: A site is an ontological figure of the instant: it appears only to disappear.

Analysis: A site is momentarily a hyper-set. As such, a site is non-well-founded. Under this definition, the momentary maximal appearance of a hyper-set constitutes an event. When a site dissipates, the transcendental order of a well-founded world is returned to ontological normalcy.

On the other hand, the pseudo-dialectic turn is marked by the occurrence of a silent singularity. This inverts the algebraic laws of the transcendental into the coalgebraic laws of the cotranscendental. This cotranscendental order endures under the sign of the symptom until the *sinthôme* terminates at the moment a nominal truth is uttered. This condition is in stark contrast to Badiou's assertion: "The formal laws of the transcendental, forced by the strong singularity, are restored as soon as the site has been dissipated …"

* * *

To understand how the notion of a greatest fixed point has a bearing on the development of the theory of mutant automata and the *sinthôme*, we need to conceptually shift the idea of a greatest fixed point from well-founded and non-well-founded set theories to the category of classes.

Let a fixed point of a *B*-transcendental be the singleton {*}, which we shall mark "**1**" and consider it to be an object in the functor category **One**. The functor category **One** is ranges over the enormous category of the proper classes of sets, [4] which we denote **Class**. Defining the functor **One**: **Class** $\rightarrow$ **Class** $\times$ **Class** allows us to express the singularity **1** in **Class** as a terminal object in the category of terminal coalgebras. Think of **Class** as the category of ontological singularities and **One** as a functor category ranging over **Class**. The functor **One**: **Class** $\rightarrow$ **Class** $\times$ **Class** establishes homeomorphisms between singularities as source objects and the intersection of ordered pairs of singularities. This gives us formal ground to treat both the event and the turn as the terminal objects of the category of coalgebras. In category theory a functional map (homomorphism) has an initial (or source) object as its domain and a terminal (or target) object as its codomain. The analogy to set-theoretical functions is easily demonstrated.

In the literature, one of the earliest applications of coalgebra concerns deterministic automata described as set of states. A next-state function dynamically ranges over states.

[4] The idea of the functor category **One** formally reiterates the discussion of proper class categories introduced in (Aczel 1988, 113; Adámek 2005, 158)

The idea of a set of states of situations in **Set** can be expanded to include a dynamic we will add a dynamic next-state trace function that moves deterministic automata between states. To this model we add the adjective "final."

Keeping in mind that a state of a situation is a power set function in **Set**. The state of a situation represents everything that is presented in that situation (except when the situation becomes an evental site). If we denote the set $\mathfrak{V}$ (*die Vertretung*)[5] as the set of everything represented (*sich vertreten*) in a situation, and assign Σ to a set of inputs on $\mathfrak{V}$ – think of inputs as the initial objects of a homeomorphism – then the next-state function **`go-on`** : $\mathfrak{V} \times \Sigma \to \mathfrak{V}$ plus a predicate **`final`** : $\mathfrak{V} \to$ **Bool** (where **Bool** is the category of (classical) Boolean algebras or its equivalent *B*-transcendental order) leads us closer to a formal account of a viable post-evental state of representation. This operation not only allows us to make our "state" $\mathfrak{V}$ the greatest fixed point of an algebra, but under the predicate **`final`** it also allows us to assign the label **1** as the termination of a $\mathfrak{V} \to$ **Bool**. Cf. (Adámek 2005, 158).

---

[5] "$\mathfrak{V}$" is the Wittenberger form of the roman type form "*V* ". This use of an old German type face plays to Gödel's extensive use of old German type forms to represent mathematical entities. On the other hand the play is complicated by Gödel's use of the sign of the universal class "*V*," [i.e. $(x)x \in V = \{x\}$] which is a roman type character (Gödel and Brown 1940, 8). Categorially speaking, *V* may denote an object of our super-large category **Class**. What is more important however, our use of "$\mathfrak{V}$" linguistically plays on the dative form of the Gewrman substantive *die Vertretung* (political representation) and its conjugation to the verb *vertreten*. German substantives are capitalized on the first letter while verbs are not. The use of the capital type form suggests a move of the "letter" from a static substantive order to a dynamic mode of standing-in-for. In this way of thinking each individual state of a set of possible states is denoted by a lower-case form. e.g., $v \in \mathfrak{V}$.

For badiou, anything presented in a situation is already-stood-in-for, or represented, by its corresponding state – except for the moment of the event. For Heidegger, it is impossible for anyone to self-present in a world because any self is already represented there, stood in for, subjugated in the place of the other by the possibility of the infinite substitution of a self lost in the universal "they," Heidegger's *der Andere*. Heidegger: "*Im Gegenteil, jeder Andere kann sie vertreten*" *weil* "*Jeder … der Andere* [*ist*] *und Keiner er selbst*." (Heidegger 2006, 126, 128). See also (Fynsk 1993, 30).

However, at the point of death no *other* stands-in for the self: "*Der Tod ist die Möglichkeit der schlechthinnigen Daseinsunmöglichkeit*. *So enthüllt sich der* Tod *als* die eigenste, unbezügliche, unüberholbare Möglichkeit. *Als solche ist er ein* ausgezeicgneter *Bevorstand* (Heidegger 2006, 250-251). On the other hand, for Badiou, death is simply the sum-zero coefficient of existence (Badiou 2009a, 269). As we shall see, nothingness and death may hold the same absolute lower bound existential value of zero. However, under the name of "nothingness," a subject-trace may terminate at algebraic points well in excess of zero.

Categorially linking a dynamic coalgebra to **Set** also allows us to give $\mathfrak{B}$ a power set treatment, which will bring our reasoning into line with the Cantorian idea of the "transfinite measure" and Badiou's notion of the "errant power" of a state of representation. Our predicate **`final`** will be assigned to the object "terminal" **1** of the category **One**, the later is equivalent to the category **Class** of set-based singularities.

The "turn" from the algebraic transcendental ordering of worlds to their cotranscendental duals, will give us a mechanism to coalgebraically order the "affectivity" of Maria Wyeth's inner world. In a coalgebraic world, under the sign of the *sinthôme*, we can invert the standard Cantorian reasoning that would otherwise render Maria's "nothingness" as being illegitimately reasoned. The inductive constructors that ultimately lead us away from Cantorian reasoning, the excesses of trans-infinitude, ontological aporia and the dead-end that is Beckett's void.

Bodies

In the preface to *Logics of Worlds* Badiou introduces the conceptual relationship between the evental trace and a 'subjectivizable' body. The latter is retroactively formed by the evental synthesis of ontological matter and existential appearance. In the post-evental world, both matter and its appearance are linked by their mutual relationship to the trace. Badiou instantiates this idea in a discussion of the evental revolt of a handful of slave gladiators, who under leadership Spartacus, in 73 BC rose up against their masters and demanded freedom. In so doing a new revolutionary body was formed. This new independent collection of revolutionary slaves, having broken from the the main 'body' of gladiator slaves, form a new cohesive body rather than breaking off into disparate 'packs' and bolting. Yet Badiou immediately questions the evental *subjectivation* of this body and its relationship to the symbolic trace of an event. For Badiou, the question is simple: what is a subject? If the trace of the event of the uprising is the symbolic force of ‘freedom,’ then the assertion ‘We slaves, want to return home,’ is the now wholly related to the transcendental synthesis of the material body of the collective and its maximal of appearance in the revolutionary world of the uprising. Badiou:

> We have a trace of the event and we have a body. Is the subject the ‘subjectivation’ of a link between the physics of the body and the name (or trace) of the event?” (Badiou 2009a, 51)

The answer to this question is both yes and no. On the one hand the strategies to escape and avoid recapture are bannered under the “subjective form borne by the body, which is determined by the statement ‘We slaves want to return home.’” (ibid.) But this assertion is linked to the symbolic occlusion of capture and incarceration, and the formation a world of escape and freedom. The world of flight holds new perils. The new revolutionary ‘body’ moves in a new present in which the signifying elements forming the body are no longer that of being slaves. Their symbolic status has inverted. The logic of the world predicated under the sign of capture is now predicated under its logical dual: *freedom*. In other words, the constructible logic of the situation which seemed unassailable before the revolt, has in an instant, inverted to the dual logic of its effect. This new logic is no longer the static logic of the situation as it was before the event. The logic of the world of freedom and escape is dynamic and mutable. The world of freedom is no longer the one that was determined by the certainty of incarceration. It is one of new possibilities and unknown consequences.

This new ‘revolutionary body’ together with the logic of its trace is characteristically cotranscendental and the relationship of subject to trace is one of bisimilarity. Point by point, the subject-body navigates an unstable mutable new world of test, choice and the confrontation of the unknown states “constituted by the consequences drawn day after day from the event’s course, that is from a principle *indexed to the possible*.”

A dynamic state process of pulling together the disparate threads of meaning in a situation, the certainty of which is no longer comprehendible, is indexed by the *possible-*

next functionality of a subject traced out by the *sinthôme* in the cognitive space of an abstract imaginary body.

For Badiou, the retroactively reconstituted synthetically elevated body of the event, is distinctly marked by the symbolic efficacy of the evental trace. This body is not the imaginary body formed by the protean flows of the *sinthôme*.

Bodies and Surfaces

The closure of a world under ontological finitude and the possibility of its 'immanent' topological opening is, according to Badiou, "… the proper concept of infinity as it is applied to worlds" (Badiou 2009a, 310). The relationship between an ontology that deems a world to be "operationally closed" under finitude and simultaneously "open" as the *smooth* topology of a site, is the "paradoxical property of the ontology of worlds – their operational closure and immanent opening." (ibid.) There is some technical theorizing behind Badiou's idea of a paradoxical opening and closing of a world because a site is schematically a topology of open sets and covers. In *Logics of Worlds* the atomic logic of a site is a geometric algebra of open sets and covers, whereas the ontology of a site in *Being and Event* is schematically presented as a collection of discrete ordinal sets, which are neither open nor closed. Grothendieck topologies axiomatize the notion of an open cover which are collections of open sets. [6] The operation of a transcendental functor (Badiou's term) is a conceptual device which traces the locally defined existential values of appearance and "attaches" them to the open sets that order the materialist base of a site. The mathematical operation that achieves this is often referred to as the functor category of *sheaves* over a topological space. An open set is different from its normal counterpart, inasmuch as open sets have a *fluidity* that normal sets do not.

---

[6] For a formal description of open sets and covers see the Appendix at the end of this dissertation.

The ontology of *Being and Event* is based solely on the construction and inductive ordering of discrete sets, whereas the objects and points of a site, under a Grothendieck regime, are open sets.The "ontologico-existential" treatment of a world, its situations, sites and traces, remain that of an initial inductive algebra, whereas the logic of the *sinthôme* is coalgebraic and coinductive.

Describing the topology of a site by extending Badiou's basic notation, the object $(A, \mathbf{Id})$ in a world, is a collection of open sets A (a cover) and a function **Id** on the real line interval $[\mu, M] = \{x \in \mathbb{R} : \mu \leq x \leq M\}$ (where $\mathbb{R}$ denotes the infinite set of the real numbers and μ and M are minimal and maximal degrees of appearance respectively). This means that for every relation $\mathbf{Id} \subset A \times A$ there is an ordered pair $\langle a, a' \rangle$ of points $a, a' \in A$, which is an element of the relation **Id**. For any two points $a_1, a_2 \in \mathbf{Id}$ then $\langle a_1, a_2 \rangle \in \mathbf{Id}$. It is common to describe a binary relation using the convenient inflex notation $a_1 \,\mathbf{Id}\, a_2$ instead of $\langle a_1, a_2 \rangle \in \mathbf{Id}$).

In the category of sheaves, a transcendental functor $T : \mathbf{Top} \times \mathbf{Top} \rightarrow \mathbf{Exist}$ gives a topological projection map $t \in \mathbf{Top} : \langle a_n, a_m \rangle \rightarrow [\mu, M]$ which associates an existential coefficient of intensity x, such that $\mu \leq x \leq M$, with each ontological pair of multiples in A. Let the terminal object of the functor T be an algebra. This treats the transcendental relation **Id** as equivalent to the binary relation $\leq$ but with a continuous, non discrete character. A function is continuous or smooth if the points in its domain or source that are *near each other*, can be mapped to points that are also near one another in the range of the function. The idea of "nearness" or "closeness" of points in a set is needed to expand and generalize the smooth topological continuity of a space. We shall see that the idea of the signifying surface of a subject body requires that the *algebraic surface of the body be smooth*. The signifying or naming processes, the *suturing* of sign to surfaces, is established when the body reiterates the continuous dynamic space precipitated by the event.

The atomic logic of a site is that of a continuous space of open multiples in the category of Grothendieck topoi, which entail certain functor processes. However, "open" is not the opposite of "closed," as classic logic reading would suggest. Open sets are the "complement" of closed ones, not pure logical negations. This has a significant bearing on why the algebraic treatment of situations that become sites is intuitionistic and not classical. This also signifies the radical structural evolution that separates *Being and Event* form *Logics of Worlds*. In a topological space the complement of an open set is closed. The negation of an open set is the *interior* of its complement, not its negation. The double negation of an open set is not necessarily equal to the open set itself. [7] Remember, in classic logic, the law of double negation states that the double negation of a proposition is materially equal to that proposition. Thus the algebra of open sets is not classical and Boolean but intuitionistic and Heyting. The intrinsic logic of a site is intuitionistic.

Smooth Bodies

An abstract subject-body, which has been fully turned by the occurrence of a silent singularity, can be schematically formed as a smooth abstract coalgebraic surface. The suture of signifying elements to surface is inscribed by the action of a mutant automaton. In this configuration, there is no "re-tying" (sic) of the "Borromean effect," nor an insurrection that causes "the One of a different kind," to "algebraically" emerge from one dialectic plane to another (Badiou 2009a, 243-244). On the contrary, in my recasting of the non-evental occurrence of a weak singularity as a "silent" turn, the smooth topology of the site (weak singularities also constitute sites) is extended in the continuity of the trace, I call *sinthôme*. In this configuration, the dynamic range of the processes that suture sign to surface in the formation of the subject, are terminal coalgebras, not the initial

[7] For a formal description of the interiors and compliments of open sets and covers see the Appendix at the end of this dissertation.

algebras that permeate *Logics of Worlds*. The suturing processes continue, in real time, until the mutant automaton that drives the formation of a subject-body, neither "cuts" nor "links" the *sinthôme* to form a "new class State, organizing a mass consensus," as it might under an algebraic configuration of a "loud" revolution. Under the sign of the *sinthôme*, the formation of a subject-body continues under the silent processes of becoming, that have been triggered by the turn. The unconscious drive of a "possible next" command hidden deep in the program registers of a mutant automaton, such as Maria Wyeth's unconscious. It is important to observe, that the formation of a body that emerges from either the "silent" or "loud" coming-into-being of a singularity, does so in open continuous space and in real time.

In point-set topology, such as the topology of a world we see in *Logics of Worlds*, the collection of sets of points and elements that comprise the multiple being-there of that world are still *of* that world (Badiou 2009a, 370). This means that there is no intensity of appearing that lies 'beneath' nor 'above' the closed interval $[\mu, M]$ of a transcendental, which closes under minimum and maximum degrees of appearance.

Under a set-theoretical configuration of a world, no multiple can exceed itself in situations that are not sites. However, "[a] world remains globally open for every local figure of its immanent composition" (ibid. 310). Thus, if a world were finite then every being which enters into the composition of the world would itself be finite. "If any infinity possessed an infinity of elements, since these elements are also of the world, the world would have to be infinite." (ibid. 308) Our ontological configuration of a world is Boolean (note: all Heyting algebras are Boolean but not conversely) yet it is faithful to the complete ontology of *Being and Event*: no world exceeds itself and is therefore closed to that which may lie outside it. This is the ontology of complete set theories and the normal spaces; in other words, this world picture falls under the closed category **Bont** $\times$ **Bont**.

The category **Bont** is the product category **Set** $\times$ **Rhet** whose categorial objects are ordered pairs of sets and meta-ontological functions. The conceptual construction of the category **Bont** entails the conjoining of *ontological* set objects and meta-ontological interpretations of them. Although this construction appears to be circular, it is one that nevertheless allows the categorial interweaving or "circulation" of the strictly mathematical ontology of sets and "modern theories of the subject and its own history." (Badiou 2003, 13) The category **Bont** is the Cartesian product category **Set** $\times$ **Rhet** which pairs neutral set objects with meta-ontological meanings that conceptually form "ontological" multiples. [8]

For the atomic logic of a world to be topologically open to any set that may be included in that world, requires that the world be ontologically composed of open sets. In point-set topology, open sets are used to distinguish between points whose proximity to each other is more important than the distance between them. Hence the notion of smooth topological space. The degree to which any two points can be separated can be specified. The idea of a relation between two points that are marked by their degree of identity, underlies Badiou's theory of appearance. This can be understood as a fluid continuity between points and the collection of points of the world. This opens the possibility of real, non-discrete treatments of the topology of a world. In this sense there is an immanence to a world which is always open and always mutable, but the possibly infinite combinations of the ways in which pairs of points can be related is always circumscribed by the ontological and algebraic parameters under which a world closes. This does mean that if a world is topologically construed as an abstract surface of open sets, then there is a sense of "smooth" infinitesimal immanence that is real, for example, under the functor category

$$F\uparrow: \mathbf{Exist} \rightarrow \mathbf{Set} \times \mathbf{Exist}$$

---

[8] The term "Bont" suggests the mildly satirical contraction of "Badiouian ontology" and "Boolean ontology."

In the category of topological spaces **Top**, the arrows between **Top**-objects (open sets) are continuous. The morphisms between topological spaces are "smooth" functions that can map non-discrete points *close together* to other non-discrete points also close together. Because such mappings are continuous, it makes sense to think of the opens sets of the topology of a world as being existentially indexed over the open subsets of real space under $\boldsymbol{F}\uparrow$. This means that the transcendental indexing of the maximal and minimal coefficients of appearance which existentially close a world are in a real sense infinitesimally (immanently) open. In symbols: $[\mu, M] = \{x \in \mathbb{R} : \mu \leq x \leq M\}$ is the real line interval from μ to M that closes the transcendental of a world. Such a world is existentially open when the open atomic multiples that make up the world are indexed to the set of real numbers $\mathbb{R}$. Let us call the indexing of the open subsets of a world to a real interval, the *immanent measure* of that world.

The idea of a real interval expanded our existential treatment of the open subsets of a topology to infinitesimal existential measures, then it is plausible to let the atomic (ontological) measure of a world also be indexed to the set of real numbers $\mathbb{R}$. Given a bijection $f : \omega \rightarrow \mathbb{R}$ we can use $\mathbb{R}$ to denote the power set $\mathcal{P}(\omega)$ because as we saw in Chapter One, the relevant issue here is the cardinal measure of a world and $\mathcal{P}(\omega)$ is that measure. Since we know $\mathbb{R}$ has a cardinality that is larger than ω_0 and we cannot construct any set of intermediate size, then it is plausible to conjecture that the cardinal measure of $\mathbb{R}$ is ω_1, which is exactly what the transfinite continuum hypothesis states. It is plausible to suggest that the "internal" topological measure of a world is that of an infinitesimally smooth space indexed to points x on the real interval $[\mathbf{0}, \mathbf{1}] = \{x \in \mathbb{R} : \mu \leq x \leq M\}$. On the other hand, the external trans-ontological measure of a world $\mathbf{m}$ is $|\mathbf{m}| = \mathbb{R}$. That which infinitely lies outside a world, as well as that which infinitesimally lies inside it, we shall call the *real measure* of that world. Like the symbolic, imaginary and real orders that may be knotted together by the Borromean trace of *le symptôm*, the ontological and existential measure of worlds can equally be knotted under the post-evental trace of the *sinthôme*.

The Cotranscendental Ordering of a World

The Pseudo-Dialectic Turn

Toward the end of Book V of *Logics of Worlds* (in the section with the evocative title "Formalizing the Upsurge?") Badiou extrapolates the formal properties of the real change that transpires in a world in which an event happens. With a razor sharp incision, Badiou severs the subject from the pure ontological constraints of a Cohen forced world bound by the mono-dimensional trajectory of set theory. In *Logics of Worlds* the formal structure of an event sutures the cut between a pure ontological materialism and an existential theory of subjective realism. The idea of burgeoning subjectivity, once *forced* to germinate on the hard ontological ground of pure being, now falls on the arable onto-logical ground of being-there. Badiou:

> Under the names of 'singularity', 'event', 'point' and 'body', from now on it will be a question of what is neither being nor appearing, neither [set-theoretical] ontology nor [algebraic] logic, but rather the aleatory result of what happens when appearing is unsettled by the being that it localizes. We pass from the theory of worlds to a theory of the support of subjects and the becoming of truths. This means that the formalization of the concept, even if it persists in borrowing resources from establishment mathematics, can no longer enjoy its previous deductive continuity; it tends to focus on formulas or diagrams whose fixation on the page does not chiefly aim to impose a demonstrative constraint, but rather to distance the concept from the ambiguities of interpretation, and to deliver it bare – according to the power of the letter alone – to its absence of sense, through which it makes truth of relation. (Badiou 2009a, 390)

This puissant passage is remarkable for a number of reasons: the passage signals a shift away from describing the singularity of being in discrete set-theoretical terms. In so doing, Badiou puts forward a theory of being and appearance which coalesces a deductive ontological materialism and a subjective existential formalism. In Badiou's configuration, the intensity of appearance of the ontological aporia he call singularities, may be localized in a real-valued topos – "according to the power of the letter alone." However, into this "onto-logical" coalescence, a dynamic thread of "truth" can be woven. This thread stitches [9] point by point, the trajectory of a relation, which I will later construe as a coalgebraic bisimulation. As such Badiou's idea of the localization of truth is elided and the corresponding post-deductive retroactive relation of truth will be transformed into the mutable suturing of the letter – a name – to the subjectivizable trace of *going-on*. In this new going-on, the generic effect of a silent singularity is already embedded in the coinductive mutability of the *sinthôme*. The formal starting point of a coinductive trajectory of the *sinthôme*, is the terminal object of an algebra, which is simultaneously the initial object of a coalgebra.

Because in Badiou's reading of an event there is no pseudo-dialectic turn involved in the sublation of the post-evental figure of an inexistent, any "existential absolutization" and "reobjectivation" of the proper inexistent of an object in a world, that "retroactively [appears] as a (new) objectivation of the site." (ibid. 394) the idea of a subjectivizable trace must be rewritten under a cotranscendental order. By Badiou's reasoning, this would mean that the post-evental world in which the reobjectivation of an inexistent is conceptually bound to the same deductive world-order, is materially grounded in the atomic logic of a world that has remained undisrupted by the event.

Any coalgebraic reading of the turn involves the revision of some of the key ideas discussed in Book V of *Logics of Worlds* (ibid. 389-396). Such a revision needs to formalize the intuitive concept of a non-sublated silent singularity. This process entails the cotranscendental ordering of a world and the conceptual coalescence of the *sinthôme* and the pseudo-dialectic turn.

[9] Badiou links the metaphor of symbolic stitching to Lacan's image of the "quilting point" or "anchoring point" expressed in the French term *point de capiton*. The term also literally refers to an upholstery button. In his seminar *Les psychoses*, (1955-1956) Lacan suggests that the *points de capiton* are points at which the "signified and signifier are knotted together."

Reforming the Transcendental

As we work towards a precise formal definition of a transcendental of a world as it is conceived in *Logics of Worlds*, it will be useful to rewrite Badiou's concise formal description of a transcendental in mathematical terms that are conceptually closer to the categorial logic that I tend to use. For brevity, I presuppose the reader is familiar with the formal description of a transcendent order of a world that Badiou extrapolates in *Logics of Worlds*, particularly Book II *Greater Logic, I. The Transcendental* (Badiou 2009a, 99-190).

Badiou:

> The concept of transcendental is without a doubt the most important operational concept in the whole of the Greater Logic [Books II, III and IV], or the theory of appearing. It designates the constituent capacity of every world to assign to what abides there, in that world variable intensities of identity … In short, 'transcendental' designates that a world, in which pure multiplicities appear in the guise of objects, in a network of identities and differences that concern the elements of *what* appears.

The following schemata involve a contemporary mathematical unpacking of Badiou's algebraic notion of the transcendental order of a world. Although the semantic and syntactic structure of these schemata differs considerably from Badiou's formalism, they are nevertheless structurally equivalent. The notions of a product and co-product object, the supremum and infimum values and the greatest and least fixed points of a transcendental order all play an extremely important role in my discussion of a cotranscendental order. A cotranscendental order of a world is the structural reversal of the transcendental order of that world.

Partial Orders

Let the transcendental T of a world **m** be the ordered pair $\langle S, \leq \rangle$, where "S" denotes a consistent ontological situation and "$\leq$" denotes an order-relation on the elements of S (we have seen how this structure also describes a B-transcendental). Presume $S \neq \varnothing$ with states $s, t \in S$. As we have seen, Badiou makes an important distinction between the existential (algebraic) order-relation of the intensity of appearance of the elements of objects in a world, and an ontological object-relation between the objects themselves, it will suffice to this consider the elements of S to be ontological entities. The order-relation $\leq$ ranges over S. With a specific treatment of S, the ontological and (greater) logical properties of the transcendental can be used to algebraically enhance the otherwise primitive ordinal relationship between any two elements of the situation. This allows a purely ontological understanding of being to be expanded to include the idea of *being-there* in a world.

In general we define an order-relation R on S as the set $R \subseteq S \times S$. This means that for any two states $s, s' \in S$, the ordered pair $\langle s, s' \rangle \in R$. Using inflex notation, can be expressed as the order-relation sRs'. This relation is:

(i) *reflexive*, if for any state s the order-relation sRs holds

(ii) *transitive*, if whenever sRs' and $s'Rs''$ holds, then $s''Rs'$ also holds

(iii) *antisymmetric*, if whenever sRs' and $s'Rs$ then $s = s'$

(iv) *trichotomous*, if for every $s, s', \ldots \in S$, either the order relation sRs', $s'Rs$ or the equivalence relation $s = s'$ holds in R

The order-relation R is said to be a *partial order* if the schemata (i), (ii) and (iii) hold. If schemata (i) and (ii) but not (iii) and (iv) hold, then R is said to be a pre-order of S. If all the schemata (i)-(iv) hold, R is said to be a total order. [10] For brevity, we may refer to the ordered pair $\langle S, \leq \rangle$ as a *poset*, when $\leq$ is a partial order and S is a set.

Substituting $\leq$ for R, the relation $s \leq s'$ can be interpreted as the sentence "s' is either greater than, or equal to s." Obviously, s is either less than or equal to s' under this notation.

In a pre-order $\langle S, \leq \rangle$, a "product" $s \times t$ of states $s, t \in S$ ("conjunction" in Badiou's nomenclature), when it exists, is defined by the properties:

(i) if $s \times t \leq s$ and $s \times t \leq t$, then $s \times t$ can be thought of as the *lower bound* of s and t

(ii) if $u \leq s$ and $u \leq t$, then $u \leq s \times t$ i.e. $s \times t$ is "greater than" any other lower bound of s and t.

This means that $s \times t$ can be thought of as the *greatest lower bound* (g.l.b.) of s and t. $s \times t$ may also be thought of as the *infimum* of S. In a poset, the *greatest lower bound*, when it exists, is unique and will be denoted $s \sqcap t$ (although in classical Boolean algebra this is often denoted $s \wedge t$, with its obvious analog in the logical operator "conjunction" of the propositional calculus).

The dual notion of a product is *co-product*. In a pre-order $\langle S, \leq \rangle$, a "co-product" $s + t$ of states $s, t \in S$ ("envelope" for Badiou), when it exists, is defined by the properties:

(iii) if $s \leq st$ and $t \leq s + t$, then $s + t$ can be thought of as the *upper bound* of s and t

(iv) if $u \leq s$ and $u \leq t$, then $u \leq s \times t$ i.e. $s \times t$ is "less than" any other lower bound of s and t.

---

[10] This differs from Badiou's distinction of a total order. Cf. (Badiou 2009a, 592-593)

The co-product $s + t$ can be thought of as the *least upper bound* (l.u.b.) of s and t. Sometimes I will refer to this as the *supremum* of S. In a poset, the l.u.b. when it exists, is unique and will be denoted $s \sqcup t$ (although in Boolean algebra is often denoted $s \vee t$, which also relates to the dyadic propositional operator "disjunction."

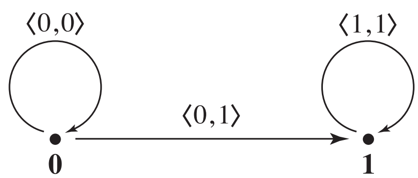
Terminal Objects: the categories of One and Two

The least upper bound of S is the union of all its elements while the greatest lower bound is their intersection. A poset $\langle S, \leq \rangle$, in which every pair of elements $s, t \in S$ have a least upper bound (envelope) and a greatest lower bound (conjunction), often called a *lattice* in the literature, is a closed algebra and is structurally equivalent to Badiou's notion of a transcendental. The product and co-product of a transcendental are categorial equivalents of the algebraic concepts of the supremum and infimum of pairs of elements of a partial order. Categorially defining the notions of the "envelope" and "conjunction" of a transcendental is formally compatible with the categorial treatments of the greatest and least fixed points of the terminal coalgebras, which will be discussed later.

We saw in Chapter One that the Boolean-valued models of Zermelo-Fraenkel set theory, under which certain forcing conditions were derived, closed under the least and maximum values $\mathbf{0}$ and $\mathbf{1}$. The definition of an upper bound limit $\mathbf{1}$ was reformulated as the generic generic ultrafilter ♂. As such, a Boolean-valued countable transitive model M^{B} (where B is some atomless Boolean algebra) of set theory is really a poset $M^{\mathbb{P}}$ where the B-transcendental partial order $\mathbb{P}$ was a Boolean algebra $B \setminus \{\mathbf{0}\}$. To say that some forcing condition $p \in \mathbb{P}$ forces an ϕ (I denoted this $p \Vdash \phi$), may do so only when $p \neq \mathbf{0}$. Thus $\mathbf{0} \Vdash \phi$ will always be true because no filter (nor ultrafilter) contains $\mathbf{0}$.

With only a greatest and a least element, a B-transcendental order is an instance of what Badiou calls the **Two**, but in the guise of the **Set**-categorial object $\mathbf{2}$. As such, a transcendental of a world can have an ordinal count no greater than 2. Any degree of difference between points $0 \leq$ and ≤ 1 are the infinitesimal real numbers between 0 and 1. Recall that the set-theoretical (ontological) structure of the discrete ordinal count of a situation, is inductively deduced from the void. The set whose only member is the void set, makes up the ordinal count of one. The set two is simply, the set whose only members are the void and the set whose only member is the void, i.e. $\mathbf{2} = \{\varnothing, \{\varnothing\}\} = \{0, 1\}$. The ordinal $2 = \{0, 1\}$ has a natural ordering $0 \leq 1$ which makes it into the pre-order **2**.

Giving a categorial interpretation, the category **2** has two objects, three arrows and a diagram:



Taking the categorial objects **0** and **1** to be the numbers 0 and 1, the initial object of the arrow $\langle 0, 1\rangle : \mathbf{0} \rightarrow \mathbf{1}$ is **0**, while its terminal object is **1**. We should note that this way of thinking is purely algebraic. The coalgebraic dual of the arrow $\langle 0, 1\rangle$, is merely the reversal of the arrow, in which the terminal object of an algebraic *transcendental* category is the initial object of a *cotranscendental* category, i.e. $\langle 0, 1\rangle : \mathbf{0} \leftarrow \mathbf{1}$. This inversion, together with the ideas of initial and terminal objects together with the inversion of the transcendental of a world, will prove to be crucial and foundational when the idea of a cotranscendental is formally described. As we develop the idea of a pseudo-dialectic turn, which coincides with the appearance of a silent singularity, the inversion of a transcendental order that is an initial algebra, lays ground for what we will understand to be as the *coexistential* emergence of the primal trace I call *sinthôme*.

In the meantime, we need to set in place the idea of an onto-logical object that is understood as not merely being the ontological analog of multiple-beings in a situation, it also expresses the existential correspondence to being-there. This may be achieved by noticing, as Badiou does, that an object in a world must have an appearance identity intensity that is determined by the bi-relational correspondence of one ontological object to another. Notice that the arrows $\langle 0,0\rangle : \mathbf{0} \to \mathbf{0}$, $\langle 0,1\rangle : \mathbf{0} \to \mathbf{1}$ and $\langle 1,1\rangle : \mathbf{1} \to \mathbf{1}$, in the above schema, are pairs of values with the categorial objects **0** and **1** as their initial and terminal referents. Clearly

$\langle 0,0\rangle = \mathbf{Id_0}$ is the identity arrow on **0** and

$\langle 1,1\rangle = \mathbf{Id_1}$ is the identity arrow on **1**

$\langle 0,1\rangle = \mathbf{Id_{01}}$ is the monic arrow $\mathbf{0} \to \mathbf{1}$

in which $\mathbf{Id_0}$ and $\mathbf{Id_1}$ are trivial instances of the categorial identity arrow and $\mathbf{Id_{01}}$.[11] Note also that **Id** is the categorial equivalent of Badiou's identity function **Id**, introduced in Section 3 of Book III in *Logics of Worlds*. Obviously my categorial treatment differs from Badiou's algebraic one, but the treatment given here is onto-logically equivalent to Badiou's 'function of appearing.' Badiou's identity function $\mathbf{Id}(x, y)$ may be read as the 'degree of identity' in which every pair $\{x, y\}$ of elements of a multiple A corresponds to an element of the transcendental T (Badiou 2009a, 243). Given that Badiou's idea of existential identity is an identity function in which the elements $x, y \in A$ fall under the order-relation $\leq$, then their differential degree of appearing p, falls in the closed interval $\mathbf{0} \leq p \leq \mathbf{1}$. This means that the degree of identity between elements of A is maximum when $p = \mathbf{1}$ and minimal when $p = \mathbf{0}$. Thus the function $\mathbf{Id}(x, y) = p$ has the infinitesimal range $\mathbf{0} \leq p \leq \mathbf{1}$.

[11] For a full categorial description of identity arrows see (Goldblatt 1979a, 25).

Because the categorial identity arrows $\mathbf{Id_0} = \mathbf{0} \to \mathbf{0}$ and $\mathbf{Id_1} = \mathbf{1} \to \mathbf{1}$ initialize and terminate on **0** and **1** respectively, their categorial composition remains singular. It turns out that there is not much we can say about $\mathbf{Id}_{\langle \mathbf{0}, \mathbf{1} \rangle}$ except its composition with the identity arrows of its initial and terminal objects. [12] This is means that there is no order-relation over $\langle \mathbf{0}, \mathbf{1} \rangle$ that could describe a transcendental indexing of any values other than the minimal **0** and the maximal **1**.

If the notion of a transcendental order is to be stronger than having only a least and greatest element, then the description of a transcendental needs to be expanded to include other operations. These include specifically the ideas of what Badiou refers to as the *envelope* and *conjunction* of a transcendental, although I will use my own terms which are common in the mathematical literature.

* * *

A transcendental T is a particular type of poset with a least and greatest elements, **0** and **1** respectively (Badiou denotes these as μ and M). Badiou notes that the transcendental of a world is a Heyting algebra with a greatest and least element such that the range $\leq$ is the infinitesimal closed interval $[\mathbf{0}, \mathbf{1}] = \{x \in \mathbb{R} : \mathbf{0} \leq x \leq \mathbf{1}\}$ where $x = p$ and $p \in \langle \mathbb{P}, \leq \rangle$.[13]

As we have seen, a Badiouian transcendental has a greatest and least element μ and M. We will denote these as the sets **0** and **1** respectively.[14] This denotational switch will make it easier to formally describe a transcendental in normal algebraic terms. In my discussion of coalgebra and the formal properties of the *sinthôme*, the greatest and least elements of an algebra are initial and terminal objects in the category of coalgebras.

---

[12] There are only two possible compositions for the arrow $\langle 0, 1 \rangle$. They are $\langle 0, 1 \rangle \circ \mathbf{Id_0} = \langle 0, 1 \rangle$ and $\langle 0, 1 \rangle \circ \mathbf{Id_0} = \langle 0, 1 \rangle$. Obviously $\mathbf{Id_0} \circ \mathbf{Id_0} = \mathbf{Id_0}$ and $\mathbf{Id_1} \circ \mathbf{Id_1} = \mathbf{Id_1}$.

[13] This description considers $\mathbb{P}$ to be a set of states in the partial order $\langle S, \leq \rangle$.

[14] The greatest and least points **0** and **1** of an algebra are denoted in a bold typeface. Their ordinal equivalents, 0 and 1, are denoted in regular typeface.

However, at the moment my discussion is restricted to algebras and coalgebras in a non-categorial form.

Every pair of (infinitesimal) points x, y between **0** and **1** must have a least upper bound (product, conjunction, infimum) $x \sqcap y$ and a greatest lower bound (co-product, envelope, supremum) $x \sqcup y$. We can easily see that $x \sqcap y = y \sqcap x$ and $x \sqcup y = y \sqcup x$ (commutativity), $x \sqcap (y \sqcap z)(x \sqcap y) \sqcap z$ and $x \sqcup (y \sqcup z) = (x \sqcup y) \sqcup z$ (associativity). From the schemata (i) – (iv) above, we can easily see that $x \sqcup 1 = 1$, $x \sqcap 1 = 0$ and $x \sqcup 0 = x$, $x \sqcap 0 = 0$ and finally $x \leq y$ if and only if $x \sqcap y = x$ and dually for $x \sqcup y = y$.

If T were a classical Boolean partial order (or B-transcendental in my nomenclature), then it would require for every element $x \in T$ the negation $\neg x$ would be *relative* to the least upper bound and the greatest lower bound of x and $\neg x$ such that $x \sqcup \neg x = \mathbf{1}$ (excluded middle) and $x \sqcap \neg x = \mathbf{0}$. Because a site is a Grothendieck topos and a transcendental order under a Grothendieck regime is a intuitionistic Heyting algebra, any classical Boolean definition of double negation, will not hold.

In classic algebra the negation of an element is usually referred to as the complement of the element. Explaining this in category-theoretic terms, we let the transcendental $T = \langle H, \leq \rangle$ be a Heyting algebra and define the algebraic complement $\neg : H \to H$ by $\neg a = a$

$\Rightarrow 0$.[15] Then $\neg a$ is the least upper bound of $\{x : a \sqcap x = 0\}$. This means that $\neg a$ is not thought of as the negation of a but as the *pseudo-compliment* of a.

The transcendental T is a Heyting algebra. A Heyting algebra H is a lattice such that for all a and b in H there is a greatest element x of H such that $a \sqcap x \leq b$. This element is the *relative pseudo-complement* of a relative to b, usually denoted $a \Rightarrow b$. Letting the largest and the smallest element of H be **1** and **0** respectively, for any element x in H we define its pseudo-complement $\neg x$ as $x = (x \Rightarrow 0)$. We know by definition that $a \sqcap \neg a = \mathbf{0}$ and $\neg a$ is the largest element having this property. On the other hand, it is not generally true that $a \sqcup \neg a = \mathbf{1}$ (excluded middle), meaning that $\neg$ is merely a pseudo-complement, not a true complement, as would be the case if T were a B-transcendental.

Efficacious Names

Sub-object classifiers are a type of categorial subset that may be thought of as a "carriers of truth." The set of truth carriers turn out to be categorial objects by which a subjectivizable body presents a truth in a world. Followings an event, any post hoc procedure that describes a body-of-truth, in Badiou's estimation, requires a retroactive operation that involves aleatory choices. A subjective choices may bear a truth or non-truth relative to an event. There is always a chance that a truth will not hold.

[15] The use of the symbol "⇒" to denote the Heyting algebra operation of relative pseudo-complement (r.p.c.) is ambiguous. The symbol is also used in this document to denote logical implication. However, "⇒" is common in the literature denoting the binary relation r.p.c. Where there is no ambiguity, I will maintain both denotational conventions. Note also that Badiou uses "⇒" to denote the binary relation "affects." For example "'*x' really affects y'* ↔ *[(Ex ⇒ Ey) = M]* " (Badiou 2005, 392) indicates that the relationship between the elements x and y is one in which the affect of x to y is maximal, or absolutely intense. Note also that Badiou's notation for material equivalence "↔" is denotationally equivalent to "⇔" as it is used in this text.

We can describe the transition of truth from ideological to material categories, by letting there be a bijective (two-way) correspondence between a collection of what we shall call "characteristic functions" from an ontological object A to a transcendental classifying set $\mathbf{2} = \{\mathbf{0}, \mathbf{1}\}$.

In categorial terms, the idea of there being some way to make a purely aleatory choice can be partially resolved by formally establishing some function between an ontological entity A and the transcendental set **2** against which truth may be borne. The formal correspondence of truth that makes **2**, that is a function between the parts of the ontological object A and the **2**, is given by a subset $B \subseteq A$. Defining a "characteristic function" $\chi_A : A \rightarrow \mathbf{2}$. This is given by the rule "for those elements B in A give output **1** and for those not in A give output **0**..." (Goldblatt 1979, 79). This idea points to a formal description of what it means to assign a "truth value" **1** or **0** to a part of A. As we have seen, the signs **1** and **0** mark also the greatest and least coefficients of appearing in a transcendental order.

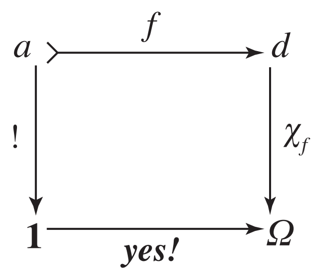
The object **2**, together with the function ***yes!*** $: \mathbf{1} \rightarrow \mathbf{2}$ provides a way to link subsets to characteristic functions and thereby give a formal categorial setting for the onto-logical passage from what Badiou calls the One to what he calls the Two. In this respect, consider the Two to be the *truth-values object Ω*.

If a transcendental T of a world is construed as a category with a terminal object **1**, a subobject classifier for T is a T-object Ω together with a T arrow ***yes!*** $: \mathbf{1} \rightarrow \Omega$ (Goldblatt 1993, §6.2 162–168). Think of Ω as a truth object.

Let $\mathcal{E}$ be an elementary site with subobject classifier ***yes!*** $: \mathbf{1} \rightarrow \Omega$. An elementary site can be schematized as a category with product objects and truth subobjects. Formally, an elementary site has other properties but these do not need to be described here. [16]

[16] For a full categorial description of an elementary site see (Goldblatt 1979, 84).

Ω -Axiom: For the T-object Ω of truth-values on $\mathcal{E}$ and a characteristic property for the arrow ***yes*** there is exactly one characteristic arrow $\chi_f : d \to \Omega$ for each monic arrow $f : a \rightarrowtail d$ [17] making the pullback square



commute. The "characteristic" arrow χ_f forms the truth character of the monic f as the subobject of $d \to \Omega$.

Conversely we can define the unique monic arrow $\mathbf{1} \to \mathbf{0}$ to be ***no!*** : $\mathbf{1} \to \Omega$. The mathematical template for this description is due to (Goldblatt 1979, 83).

* * *

An elementary site, in which an elementary body may precipitate, supports the following arrows.

Let $\cap : \Omega \times \Omega \to \Omega$, $\cup : \Omega \times \Omega \to \Omega$ and $\Rightarrow : \Omega \times \Omega \to \Omega$ be the *efficacious* arrows of an elementary site whose abstract topology is the topos $\mathcal{E}$. These arrows behave like the normal logical connectives – conjunction, disjunction and material implication – and act as the "logical operators" for the efficacious parts of the topology $\mathcal{E}(d, \Omega)$. These

"operators" define the normal logical operations by

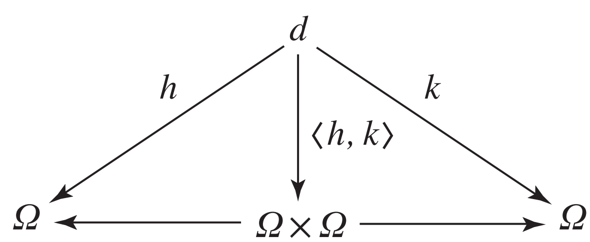
- $\mathrm{h} \cap \mathrm{k} = \cap \circ \langle h, k \rangle$

[17] Remember, monic arrows "$\rightarrowtail$" are the categorial equivalents of injective or one-one functions in which no two distinct inputs have the same output, i.e. for inputs $x, y \in A$, if $f(x) = f(x)$ then $x = y$.

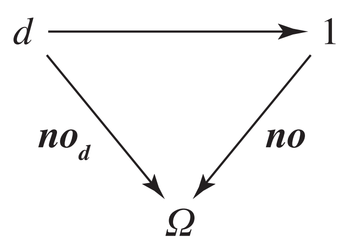
- $h \cup k = \cup \circ \langle h, k \rangle$
- $h \Rightarrow k = \Rightarrow \circ \langle h, k \rangle$

where $\langle h, k \rangle$ is the product arrow of h and k:

The structure $\langle \mathcal{E}\langle d, \Omega \rangle, \cap, \cup, \Rightarrow, \boldsymbol{no!_d} \rangle$ as defined by the diagram



is a Heyting algebra where the least element $\boldsymbol{no!_d}$ is given by the diagram



Remember, the product object in the category of algebras is the greatest lower bound point of a partial order of the algebra. The disjoint union (or sum) object of the same category is least upper bound of the partial order.

An imaginary abstract body is not grounded in atomic matter nor does it necessarily have a material surface. Such a body I will call *elementary*. The surface of an abstract body, to which names may be symbolically fixed, I will call an *elementary topos*. The parts of the body which affirm a name, which following Badiou, I will call *efficacious*. Yet an abstract body can equally bear a subjective formalism whose simplest part is the sub-object that is classified under Ω as either **1** or **0**. A name may have many senses but it has only one referent, the truth object Ω.[18]

Naming

Turning Away

In the following intertextual analysis of the protagonist subject of Beckett's *The Unnamable*, it suffices to restrict the discussion to the purely onto-mathematical properties of Zermelo-Fraenkel set theory. This is plausible because Badiou takes care to describe the idea of singularity, in both *Being and Event* and *Logics of World*s, to be that of the ontological impossibility of the aporetic self inclusion of sets. There is also no need to identify the "body" of Beckett's unnamable "I." This is because the Zermelo-Fraenkel axiom of foundation ontologically legislates that in any situation derived from consistent classical set theory must be well-founded. The normal discrete sets of the classic set-theoretical footing of *Being and Event*, are well-founded under both the axiom of foundation and extensionality. Extensionality determines that there are indeed such entities, while foundation guarantees the existence of at least one ontological entity. If

[18] German logician and philosopher Gottlob Frege (1848-1925) drew a distinction between the sense and reference of formal sentences. Given a normal interpretation of arithmetic, the referent of the sentence "2+2" is the number 4. The sense of the number 4 is the numeral "4." On the other hand, the referent of the sentence "2+2=4" is the truth value "true" (or **1** in my denotation). The referent of the sentence "2+2=5" is the truth value "false" (or **0**). Under this example and in the context of this discussion, the sense of a name is various but the referent of the name is a truth object O. Under the efficacious (or classifying) arrows ***yes!*** and ***no!*** the target object of the unique monic "efficacious" arrows is either the truth object **0** or **1**.

this were not the case, then any notion of an event would be either meaningless or redundant because all situations could simultaneously be sites. The presentation of being as being and situations that may represent the non-presentation of being ambiguously merge.

It is also plausible to note that the tyranny of reason under which Beckett's unnamable "I" finds himself, is classical. The aporetic anomalies of set theory, set against a cold war anxiety a post World War II uncertainty, are recognizably allegorical in the soliloquy of "I." On the other hand, Joan Didion's Maria Wyeth is meddled by the post-modern logic of her inner world. This logic, I argue, is typically coalgebraic.

Badiou's reading of the interminable torment in which Beckett's "I" finds himself, is undoubtedly bound by the legislative ontological notion of a well-founded universe of sets. For "I"'s going-on to cease would imply that the torment of non-being would end in a moment of ontological non-well-founded-ness: the event. Without an event, Beckett's tortured trajectory circulates around an imagined nostalgia for a silence which is haunted by the terminal certainty of death:

> I'll soon go silent for good, in spite of its being prohibited … I think I'll soon be dead, I hope I find it a change. I thought that would be my reward for having spoken so long and so valiantly, to enter living into silence … (Beckett 1958, 153)

Beckett's "I" finds no silent termination of the going-on, and there is no guarantee that death would be that place:

> I can't say it, I can't say why I should have liked to be silent a little before being dead, so in the end to be a little as I always was and never could be, without fear of worst to come … (ibid. 153-154)

Neither does Maria Wyeth deal well.

> All that day Maria thought of the fetuses in the East River, translucent as jellyfish, floating past the big sewerage outfalls with orange peels. She did not go to New York. (Didion 2005, 116)

Maria Wyeth is not dealing with the moral and emotional ramifications of her abortion. In the imaginary registers of her neurosis, the metonymy "floating orange peel" →"floating fetus" has bonded. She can give no legitimate voice to that which cannot be said. No name is allowed to stick. The exact point of Maria's turn from the Hollywood myth of material and artistic success, is indiscernible. However, we can see that a turn has occurred. The singular mutability of Maria's *turn for the worst* is indeterminable. We know that something has turned but who knows when she turned?

As Didion's narrative unfolds, we infer from Maria's soliloquy that some turn must have taken place. The saturated state of Maria's emotional breakdown has occurred and her symptoms attest to it. There is only one recognizable marker that is meaningful in the patriarchal from which she has unceremoniously fallen: material and artistic failure. In the eyes of her acquaintances, Maria has turned for the worse. Her casual dissociated sex, alcohol excess and drug abuse have replaced the moral and material certitude of hard work. Maria's successful ex-husband Hollywood director Carter Lang, works hard, is motivated and is correspondingly rewarded. Maria's acting career has failed and her emotional constitution is in tatters. In the words of Helene, Maria's one time friend but now moral adversary: "Maria has never been able to bear Carter's success." (Didion 2005, 11) Maria is not only incapable of bearing Carter's success, but in his jealousy and frantic need to maintain appearances he forbids *her* from bearing the child of another man. Carter orders her abortion. Yet like Beckett's 'I,' Maria must go on, but the going-on has become internalized and hopelessly entwined in the near catatonic thread of her solipsistic. The outward indications of Maria's condition are the signs of carelessness and selfishness typical of a emotionally wounded decadent. Maria Wyeth is a failed and corrupted member of the rich and privileged middle class of Hollywood's cracked-up *élite*.

Yet Maria finds her own way out. Like the floating fetuses, Maria somehow stays afloat. Flowing with the deep psychological currents of the *sinthôme*, she navigates an ill-formed world of floating signifiers, names, signs and markers that will not adhere to anything that may fixes mean. The big "T" in the car park of the Thrifty Mart, near which Maria meets the agent of her abortionist, becomes a name that will not properly adhere to the representational mess of indexical detritus that orders her world before the turn. In the hope that authentic meaning may clarify her ride to the abortionist she nevertheless remains lost. Maria somehow goes-on, and on and on until almost indiscernibly, there is a turn and a name sticks, which both anchors her and subsumes the intensity of her emotional grief and solipsistic vacuity: "nothingness."

Silent Singularities

Maria's has abused her actual body and she is compliant to the will of others. She has emotionally flushed any symbolic meaning of *this* body down the drain along with the remains of her aborted fetus. In imaginary registers Maria now inhabits a new abstract body, silently marked by the subtle force of whole new trace: ineffectual internalized defiance. This body has slowly emerged in barrenness of a strange new world. Maria tests her world. She tests at every turn, hoping for a sign that may bear a semblance of truth. Her life is full of lies and falsehoods. All require testing:

> "This is just induced menstruation," she could hear the doctor saying. "Nothing to have emotional difficulties about … just a little local on the cervix, there relax Maria, I said *relax*." (Didion 2005, 82).

As the lies continue she has no way of knowing which words to trust. She continues to listen and passively observe her world from the dead space of emotional overload. Somehow she continues to test the lies and the portentous signs, which if not heeded nor properly understood could lead to calamity. These signs literally flashed passed as she drives on the freeway: GO BACK, DO NOT ENTER. She continues to question, to test names against anything that may bear truth. The going-on finally ends in the calm which descends when one name finally rings true: *nothingness*. After the suicide of her friend BZ, Maria's strange, almost incidental knowledge reaffirms her place in truth. "*One thing in my defense, not that it matters: … I know what "nothing" means* ..." (ibid. 84)

Clearly, Maria's imaginary body of truth is not the one we find in *Logics of Worlds*. Maria's wholly *democratic* body is diametrically *other* than any body she could reveal to the actual world. Her imaginary body of truth is without the synthetic objectivity of appearing and being, we might expect to follow an evental epiphany. Maria's imaginary body is different from the subjectivizable body of a Badiouian event that retroactively reconstitutes under the allegorical flow of an evental trace. There are significant differences between the two bodies. Firstly: in the ways in which the post-evental body confronts the global situation of the event; secondly: in the way in which Maria supplants the "body of lies" which drove her to the edge of real madness. Both bodies ratify their traces "point by point" with a decision process of "singular choices, with decisions that involve the 'yes' and the 'no' " (Badiou 2009a, 51). The meaning of each body is ratified in the way their divergent traces are ordered. One is topological and transcendental, while the other is abstract and cotranscendental.

The world in which the post-evental body of the slaves under Spartacus, is the same one that still embodies the same hierarchical logic that existed before the revolutionary event of their escape. The expansive growth of a burgeoning body, although both armored and organized, runs the same aleatory risks that existed before the revolt-event. Facing the real possibility of failure and the diminution of a new invigorated body, the threat of destruction and the return to enslavement or death, they must turn to a truly *insurgent* logic.

An insurgent body needs no anchoring in the *truth* of an ideal trace: freedom. Let a body emerges from mutating non-symbolic processes that continue and fluctuate in the uncertainty of radical upheaval. Such a trace is unknowable to adversaries and indeterminable by enemies. In other words, apply an insurgent logic that is found in the final semantics of ill-founded worlds. By its definition, bisimilarity is an inconsistent (ill-founded) version of itself. There is no hierarchical order on the pairs of bisimulations. Nor is there transcendent certainty in a world that has been turned. Bisimilarity involves purely aleatory processes that are grounded in observation, not construction. This is why bisimilarity can be effectively used to reason infinite or circular logics. In this way the ill-founded logic of the event can include itself as a part of its own affect and thereby name its own impossibility.

It is plausible to consider the solipsistic "I" of Beckett – who as Badiou observes is without love – to be without the means whereby a truth can be decided: " … love begins in a pure encounter, which is neither destined or predestined, except by the chance crossing of two trajectories." (Badiou 2003, 27) However, the legislative tyranny of Beckett's going-on and the corresponding imperative to speak, remain "an imperative for the sake of the oscillation or the undecidability of everything." (ibid. 2)

To name a truth requires functional mechanisms that are hopelessly beyond the limits of the logic that order Beckett's world. Even though the going-on gets tough, Beckett's unnamable "I" fantasizes an impossible turn of events in which the nominal could escape the bonds of soliloquy through the conduit opened by a proper name: "… if they ever succeeded in getting me to give a voice to Worm, in a moment of euphory perhaps, perhaps I'll succeed in making it mine, in a moment of confusion." (Beckett 1958a, 85)

Beckett's unnamable "I" never escapes the tyranny of reason, nor the dense inertia and aporetic stagnation that characterizes his proximity to the void. "[N]o one has ever met anyone before my eyes, these creatures have never been … Only I and this black void have ever been." "A pox on void. Unmoreable unlessable unworseable evermost almost void." (Beckett 1958a, 21) His distant proximity is entailed by a logic which never escapes the constructible legacy of its empty origination. There is nothingness at the beginning and there is nothingness at the end. The representation of the silent nothingness that haunts him is forbidden under some Kafkaesque alterity that is always referred to in the third person: "them," "they" … . On the other hand, Maria Wyeth – who is also without love – makes an *adjustment* to her world with such imperceptible subtlety and psychological cunning of such radical affect, that the entire logic of her world inverts and the trace of that inversion becomes a mutable string of choices in which she deftly plays any hand that is laid before her. Maria's world is no longer constructed with the same transcendental order as the world of Beckett's "I." Her world has mutated into something else. From her absent father, Maria had already learned to play it as it lays.

Chapter THREE

Final Semantics

In Badiou's classic set-theoretical ontology a situation is a collection of presented multiples (sets), which may be ontologically consistent under the axioms, rules and proofs of Zermelo-Fraenkel set theory. Any abstract ontological investigation of the question of being, as Badiou argues in his earlier writings, can only be effectively carried out in the mathematical language of classical set theory. In his later writings, Badiou defines a site formally as the open sets and covers of Grothendieck topoi. As we have seen, under a Grothendieck umbrella the parts of a situation can be construed as the *subobject classifiers* of the "ontologico-existential" idea of localized "truth." Grothendieck's generalization of the functor categories of sheaves over topological spaces is based on the observation that certain set-based axioms are expressible as the categorial properties of open covers and truth objects in the category of smooth topologies.

A systematic discussion of sheaves and categorial truth objects is beyond the scope this discussion, nor is it required. Extensive discussions of category theory and Grothendieck topology, both introductory and advanced can be found elsewhere.[1] However, the idea of locally defined "truth," and smooth (continuous) open-cover surfaces can sufficiently

[1] Particularly (Fourman et al. 1979; Freyd 1972, 1987; Goldblatt 1979a, 1987b; Lawvere and Schanuel 1997; Mac Lane and Moerdijk 1992)

inform any discussion of an abstract body whose conceptual integrity relies on its ability to effectively fix the signs of *nominal truth* (as opposed to actual truth) to abstract surface. Grothendieck topologies are abstract geometric algebras, which entail the idea of the potential existence of objects which are only partially defined (as opposed to objects whose actual existence are totally defined). The immanent existence of a virtual object, one whose existence is only partially defined, can be given by its proximity to an actual object – call this a multiple – whose existence is totally defined. A *truth equivalence* relationship between virtual and actual objects can be established if the degree of their proximity or closeness, can be construed as an equivalence. This suggests the idea that bisimulations can express affective equivalence relationships. As we have seen, set-theoretical (or ontological) equivalence is extensional. Bisimulations are intensional. This means that the multiples which make up a situation can be cotranscendentally equivalent without being ontologically equivalent. The objects of a post-event situation do not need to have extensional equivalence to bear the same truth. Nor do these multiples need their existential degree of intensity to be equivalent. Bisimilarity, as a relationship between to two post-evental states, is a coarser than extensional equivalence.

In *Being and Event*, multiples are described as the sets presented in ontological situations. The ontological status of multiples is determined by the axioms and rules of classical Zermelo-Fraenkel set theory. The elements of each multiple presented in a situation are inductively constructed by an indexical operation which associates each multiple with a discrete ordinal number. This number is the ordinal count of the multiple. The ordinal count of a multiple is also *represented* by its *states* (*states of representation*), which correspond to the power set of the multiple. The power (or measure) of the state of a multiple (situations are also multiples) is its cardinal number. However, set-theoretical ontology is non representational in its relation to being – this relationship is purely presentational – and therefore the faculty of representation completely breaks down in situations in which being itself is presented. Such situations form sites.

The axiom of foundation – sometimes know as the axiom of regularity – does not resolve the "upper bound" paradoxes associated with the systematic axiomatization of sets determined by the predicate of set membership. However, regularity can be summoned to establish results about the well-founding of sets, as well as to establish general results associated with ordinal numbers. As we shall see, the ideas of coalgebra and bisimulation, which I used to develop a formal description of a pseudo-dialectic turn and the *sinthôme*, introduces an anti-foundation axiom which conditions the post-ontology of an event. The trace of the occurrence of a singularity – any event or turn involves the occurrence of a singularity – is necessarily cotranscendental. If a situation admits the existence, if only momentarily, of non-well-founded multiples such as the proper inexistent of a set, then the trace of that occurrence at least provides *prima facie* grounds for what we shall consider to be the cotranscendental logic of the trace of a pseudo-dialectic turn: the *sinthôme*.

The materialist dialectic evolution that turns upon an event, will do so at the highest fixed point of a transcendental: the point of the maximal appearance of a proper inexistent of an evolutionary situation forms an evental site. According to Badiou, something new emerges from an event, which constructs itself with exactly the same upward dialectic motivation that preceded the event. This new thing is a subjectivizable body onto which meanings are inscribed, or in Badiou's language, a body on which a subjectivizable formalism is borne. A point-by-point process which faithfully inscribes the truth of the event, gathers a subjectivizable body capable of faithfully bearing the logic of real change. Badiou refers to subjectivizable bodies which resist or occlude the evental trace, as reactionary and occluding subjects of an event.

A pseudo-dialectic turn does not involve the sublation nor synthesis of a material body. The imperceptible force of the silent singularity that propagates the turn, also instigates a reversal of any inductive evolution at very point the force of the singularity could have sublated it. Any upward evolution of states is reversed under the pseudo-dialectic turn, but it does so in the imaginary registers of an abstract subject. The symbolic significance of the turn are hidden by the imaginary flow of the symptom.

Under a materialist dialectic, of the sort we find in Badiou's writing, any successor-based principle of ordinal induction over states becomes impossible to count at the precise moment the count, counts itself as one. The ordinal count of any state is completely trivialized when the inductive forces of a situation are terminated under the weight of their own ontological superabundance: the One appears in a world.

Functors

In category theory, a functor operation is a transformation from one category to another, which preserves the categorial structure of the source category. For example, a functor F : **Exist**$\rightarrow$ **Set** $\times$ **Set**[2] is a homeomorphic relation between existential categories, such as the transcendental order of worlds, and the categories of the ontological structures that support the notion of pure being. Thus, the transformative functor operation F is one that can establish a condition of being-there in a world that preserves the existential algebraic structure of that world. Such an operation Badiou calls a "transcendental functor." (Badiou 2009a, 76)

The term "functor" was devised by German born logician and philosopher Rudolf Carnap (1891-1970) who first coined the term to describe the isomorphic relationships between

[2] Remember, in the category **Set**, a Cartesian product of two set objects $a \times b$, can only be defined up to isomorphism. As we have seen, a categorial Cartesian product object is based on the set-theoretical definition of the product set $A \times B = \{\langle x, y\rangle : x \in A \wedge y \in B\}$. By using a pair of projections $p_A : A \times B \rightarrow$ A and $p_B : A \times B \rightarrow$ B a unique arrow $\langle$x, y$\rangle$: $c \rightarrow a \times b$ can be obtained from a pair of projection maps ($pr_a : a \times b \rightarrow a$, $pr_b : a \times b \rightarrow a$). For a full categorial description see (Goldblatt 1979a, 46-50). The categorial dual of the product map $a \times b$ is the co-product map $a + b$ (ibid. 54). In the functor category of algebras, Cartesian product objects are equivalent to the g.l.b. of an algebraic partial order of sets, while co-product objects are the l.u.b. union of the elements in a poset.

functions and predicates in language of first-order logic. In a modern context, functors are usually seen as a transformations from one category to another that preserves the categorial structure of the source category. Unlike consistent first-order set theories in which functions can only range over sets, functor categories have other categories as well as morphisms (or arrows) as their objects and may therefore map category to category as well as morphism to morphism while still maintaining their operational integrity. We shall see how the idea of ordinary induction can be categorially reformulated as an *initiality* principle which allows a theoretical trajectory to easily and uniformly move between the various categories that are based on the principles of induction, such as **Set** and **Exist**. In this chapter I shall employ *cotranscendental* functors to describe the category of terminal coalgebras and a type of abstract transformation system that summons the coalgebra of a mutant automaton.

I have described two general categories **Set** and **Exist** (there is a convention in category theory to indicate categories with bold type). The category theory the symbol **Set** usually denotes the category of discrete sets and all the functions between them. **Finset** is the category of all sets with finite functions between them. The category **Set**$^{-}$ is the category of sets which admit an anti-foundation axiom. The category **Set**$^{-}$ presumes a set-theoretical axiom system obtained by replacing the foundation axiom in Zermelo-Fraenkel by an axiom an anti-foundation axiom.

An important category **Top** is the category of topological spaces and all the smooth (or continuous) functions between them. Strictly speaking, a transcendental functor (as Badiou conceives it) is a formal point-wise functor category of sheaves that “stitch” (or index) the existential objects of **Exist** (i.e. sheaves) to the smooth ontological base objects of **Top**, which are open sets and covers. For brevity I presume **Set** includes the category of ontological entities that are sets, which may be either discrete or smooth open sets, and all the functions between them. Although this is technically clumsy, the ambiguity simplifies my formal description of the ontological localization of existential values and the formulation of the idea of a subjectivizable body gathered under an evental trace. This formalism may then be expanded to include the ideas of a pseudo-dialectic turn, an abstract mutant automaton and the *sinthôme*.

There is a further ambiguity in my use of "**Set**." A deeper inspection of the pure set-theoretical category **Set** reveals an embedded category I call **Bont**. **Bont** is a functor category formed by all the functors between **Set** and a philosophical-linguistic category I call **Rhet**. **Bont** is obtained by assigning natural linguistic elements to set-theoretical entities, in much the such as we saw the set-theoretical sign for the empty set $\varnothing$ being rhetorically "sutured" to the proper name of being. **Rhet** can also be thought of as a meta-ontological category that imputes ontological meaning to the purely mathematical set objects that Badiou calls multiples.

* * *

The three categories **Set**, **Exist**, **Rhet** are related by the functor F : $\mathbf{Exist} \times \mathbf{Set} \rightarrow \mathbf{Lang}$ (i.e. the fibered product over categories $\mathbf{Exist} \times_{\mathbf{Lang}} \mathbf{Set}$). **Lang** denotes the category of natural languages.

Formally, the fibered product (or pullback) $B \times_A C$ is a subset of the product $B \times C$ and therefore has two projection maps π_1: $B \times_A C \rightarrow B$ and π_2: $B \times_A C \rightarrow C$ such that the *pushout* $B \leftarrow B \times_A C \rightarrow C$ commutes to the pullback $B \rightarrow A \leftarrow C$. Substituting the object categories **Set**, **Exist**, **Lang** for the category objects A, B, C, the resulting functor pullback F_f, G_g : $\mathbf{Set} \rightarrow \mathbf{Lang} \leftarrow \mathbf{Exist}$ gives the structural foundation for the incorporation of natural language categories into onto-logical topologies.

Included in the category $\mathbf{Exist} \times_{\mathbf{Lang}} \mathbf{Set}$ are two "evental" categories, which I denote $\mathbf{Event2}^{\omega}$ and **Event{A}**. These categories entail an "evental" B-transcendental functor E^{ω} : $\mathbf{Exist} \times_{\mathbf{Lang}} \mathbf{Set} \rightarrow \mathbf{Event2}^{\omega}$ and the transcendental functor $E^{\{A\}}$: $\mathbf{Exist} \times_{\mathbf{Lang}} \mathbf{Set} \rightarrow$

Event{*A*}. The *B*-transcendental functor E^{ω} describes the occurrence of a pure ontological singularity of the sort we find in *Being and Event*, while the transcendental functor **Event{A}** describes the onto-logical singularities we find in *Logics of Worlds*. A *cotranscendental functor* E^{-} can be described E^{-}: **Exist** $\times_{\mathbf{Lang}}$ **Set**→ **Event**$^{-}$ where **Event**$^{-}$ denotes the "pseudo-evental" occurrence of a silent singularity I call the pseudo-dialectic turn.

As effectively as a functor indexes ontological categories to existential ones, it can be formally demonstrated that the category of natural language objects and morphisms of **Bont** can be effectively mapped to the onto-logical category **Exist** × **Set**. This operation can be formally described under a category with terminal objects, pullbacks, equalizers and a subobject classifier such that the morphism ***two*** : {**1**} → Ω is in **Set**. The category **Set** can then be formally describe as an elementary topos.

* * *

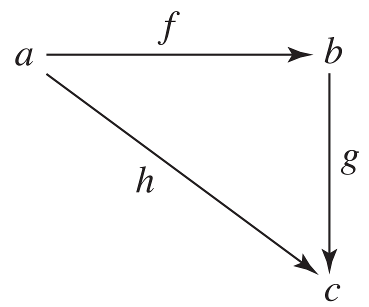
Consider the categories **Exist** and **Top**. A "transcendental" functor T from category **Exist** to the category **Top** is a function that assigns ...

(i) to each **Exist**-object a, a **Top**-object $\mathsf{T}(a)$;

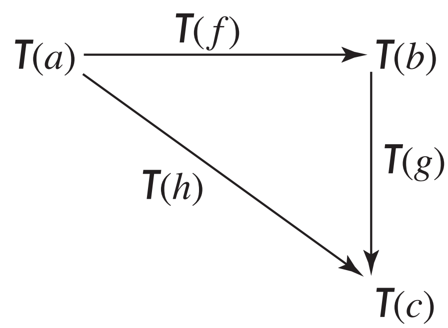
(ii) to each **Exist** arrow $f : a \to b$ a **Top**-arrow $\mathsf{T}(f) : \mathsf{T}(a) \to \mathsf{T}(b)$ can be defined such that any identity arrow on a is assigned the identity arrow on $\mathsf{T}(a)$ and ...

(iii) given two arrows g, f their composite $\mathsf{F}(g \circ f)$ is the composite of their T-images $\mathsf{T}(g) \circ \mathsf{T}(g)$.

Whenever the diagram



commutes in the category **Exist**, then the “transcendental functor” diagram



commutes in **Top**. In this way, a transcendental functor T : **Exist**→ **Top** lays the theoretical grounds for the (re)ascendence from the “transcendental synthesis of appearing back to the real synthesis of multiple being” (ibid. 289). At the same time as it preserves the existential degrees of the source category **Exist**, the functor preserves the material integrity of the target category **Top**. However, under the synthesis of appearing and being the operation of the transcendental functor fails to prevent the notion of multiple being becoming caught between the discrete ontology of **Set** and the smooth place of being located in **Top**. Badiou does not seem to notice this. Only after the occurrence of a pseudo-dialectic turn and the mutant flow of the *sinthôme* the dubiety may be sidestepped.

Final Semantics

Final semantics rose in significance in the last two decades of the twentieth century with the advances of (Aczel 1988, 1993; Jacobs 1997; Rutten 1992). The development of final semantics in the 1980s and 1990s, was largely motivated by a growing interest in the use of coalgebras and category theory in the development of abstract automata in computer science and growing interest in the binary relations known as *bisimulations*.

An equivalent of bisimilarity relations had already been developed in the various models of Kripke semantics (Goldblatt 1987a, 1993). Final semantics also provides a "rich and deep" perspective on the duality between (initial) induction and (final) coinduction. (Sangiorgi 2009b, 113).

Under the general heading of final semantics I consider both the event and the pseudo-dialectic turn to be final states. The notions of bisimulation and finality are crucial concepts I summon to formally construct a viable models of mutant automata. As Badiou points out, the post-evental conditions of a situation are radically volatile and ontologically dispersive:

> The brutal modification under the disappearing impetus of the a strong (evental) singularity of the transcendental value of $\varnothing_A$ (the inexistent of the object $(A, \mathbf{Id})$), cannot leave in tact the transcendental indexing of *A*, nor, consequently, the general regime of appearing in the world of the elements of *A*. Bit by bit, the whole protocol of the object will be overturned. A re-objectification of *A* will have taken place which retroactively appears as a (new) objectivation of the site. (Badiou 2009a, 394)

The idea of finality of the overturning of the "whole protocol of the object" as a starting point of a *new* objectivation, begs to question as to how the retroactive "re-objectification" of a post transcendental order of the post-evental world will behave after the destructive impetus of the event. According to Badiou, the laws of the transcendental that govern a world will return in force the instant the event dissipates. Badiou:

> The main effect of the appearance/disappearance of the site is the sublation of the inexistent $\mathbf{E}\varnothing_A = M$ where we from $\mathbf{E}\varnothing_A = \mu$. The laws of [the transcendental ordering of a world] T, forced by the strong singularity, are restored as soon as the site has been dissipated (ibid. 395).

The sublation of $\varnothing_A$ from a minimal to a maximal degree of existential intensity is final and the world returns to a state of normalcy.

I argue, however, that finality states such as the momentary impulse of the event, do not necessarily return to normalcy under the transcendental order of a world. According to Badiou, the maximal intensity of the evental appearance of a strong singularity, an exception, a proper inexistent, momentarily inverts the transcendental order of a world in which it ontologically non-existed, now exists with the force of total appearance – an absolute being-there. In contradistinction I argue that the sheer force of the occurrence of a singularity can so radically changes the transcendental order of the world that the ill-founded logic of the site is born of the trace. The trace inevitably bears the cotranscendental logic of finality and can totally reconfigure the meaning of a a post-evental situation that is in complete opposition to the constructibility paradigm of a transcendental order. The mutable character of a finality trace has no place and no proper functionality in a world that returns to the initial logic of a transcendental order.

In a Badiouian world the forces of normalcy close over the site but the retroactive logic of re-objectification of the site runs the risk of descending into an affective nostalgia for the event whereby no actual re-objectification takes place. If the event is to be seen as a dialectic elevation of the proper inexistent of a situation, from a minimal to a maximal coefficient of existence, then there is no certainty as to what exactly has been sublated. Unless the sublation is ratified by exactly the same logical structures of representation and appearance that were momentarily overturned onsite, the representation of the power of the event will be diminished by the flow of historicity and the flattening power of the status quo. The difference, according to Badiou, is that there is a "creative synthesis of the logic of appearing." But this is none other than the abandoned anonymous subject of *Being and Event*, returned in the form a subjectivizable body: an embodiment of truth and material sacrament. In the transition from the nothingness of ($\mathbf{E}A = \mu$) to the maximizing power of ($\mathbf{E}A = M$), a body incarnates the "creative synthesis of the logic of appearing." The "materiality of a subject of truth polarizes the objects of a world according to the generic destiny of a truth … a body allows the ontological destiny of appearing itself to appear" (Badiou 2009a, 483). A subject enters the world and takes on material existence, point by point, stitch by stitch, suture by suture. At the same time as the proper inexistent of a situation maximizes its evental appearing in a world, a body may begin to form as an onto-logical straw man, projected sheaf by sheaf, germ by germ, stalk by stalk into a world. If this is to be avoided, the logic of the evental trace must be that of a cotranscendental final coalgebra, not the transcendental initial algebra of a well-founded world. The transcendental logic of a world may indeed reconfigure itself in the moments after the event, but the spirit of exception must live on in the trace if a *faithful* subjectivizable body is to be authentically formed under its flow.

* * *

A definition of *final coalgebras* can be established in the category of non-standard sets and partial orders. Non-standard sets, as we have seen, are non-well-founded multiples such as $\varnothing_A$. Coalgebras are simple mathematical structures that generalize the notion of a greatest fixed point. For example, in a transcendental ordering of a world the maximal coefficient of appearance of a multiple is a greatest fixed point.

Importantly, dynamic observational equivalence relations, such as the *sinthôme*, can be derived from instances of a single coalgebraic definition. Such a definition is parametric to the functor of appearing in a state. Coalgebraic principles can link a name to a thing as part of the general coalgebraic flow of forces that follow an evental turn.

As we have seen, my investigation of coalgebraic form is primarily categorial. An effective categorial treatment of coalgebra can be quite elementary: given a category **Set** and a functor F : **Set**$\rightarrow$ **Set**, a coalgebra of F is a pair $\langle A, \xrightarrow{\alpha} \rangle$ where A is an object in **Set** and $A \xrightarrow{\alpha} F(A)$ is an arrow in **Set**. F is then an *endofunctor* of **Set**. Intuitively, endofunctors are "self referential" arrows whose target categories *are* their source categories. An endomap is a map in which the domain object (source) and codomain objects (target) are the same. Endomaps are not necessarily identity maps. [3]

* * *

A situation A can be *coinductively* defined if it is the *greatest solution* of an *inequation*. This means that the ontological axioms of extensionality and foundation, that held in A become compromised by a *is-not-the-same-as* relationship between the states $s, t, \ldots \in S$ that *represent* every part of A. The equation $A = \{A\}$ implies the inequation $A \neq A$. That is, if every situation counts itself as one, then the situation cannot not equal itself. This is in direct contradiction to the axiom of extensionality. Of course, this is impossible in well-founded set theories, but the idea of the ontologically valid inequation $A \neq \{A\}$ presents the "impossible truth" of an exception, which is the evental site, whereby $A = \{A\}$. A *coinductive proof principle* can determine that if any set or part of a situation is the greatest solution of the inequation implied by the "impossible" proposition $A = \{A\}$, then

---

[3] Algebras of a functor in the category $\boldsymbol{S}^{\circlearrowleft}$ are endomaps of sets. The idea of endomaps are related to bisimulations. Endomaps are self-referential in the sense that the domain of the map is its codomain. In other words, in $\boldsymbol{S}^{\circlearrowleft}$ the source objects of $\boldsymbol{S}$ are its target objects (Lawvere and Schanuel 1997, 136-138).

An identity map over sets is one in which the domain and codomain of the map are the same set A, for example, and for each member $a \in A$, the function $f(a) = a$ is an object of the category $\boldsymbol{\mathcal{S}}^{\circlearrowleft}$ of endomaps of sets.

that solution is also contained in the situation. If a set *B* is defined inductively, then it is the *least solution* of the equation implied by extensionality.

By way of contrast, if a situation *A* is defined *inductively*, it is the *least* solution of an equation. This means that in situations, the ontological axiom of extensionality is maintained by the *is-exactly-the-same-as* relationship between the elements and states of *A*, as it is in classical set theory. The states and parts of *A* that are inductively well-founded under the axioms of set theory and the foundational elements of *A*.

Sinthôme: a turn from the worse

The notion of *sinthôme* has emerged in the writings of Israeli-born psychoanalytic theorist and artist Bracha Lichtenberg Ettinger. Ettinger's reworking of "Lacan's return to an ancient term which he uses to describe the symptom" [4] presents the *sinthôme* as a shared mysterious trace of an "*Unheimlich*[*e*] ... *objet-a*". In Ettinger's estimation the *sinthôme* is a trace that emanates from "where the [death] drive and desire meet the Thing on the screen of fantasy, … where transgressive psychic real *things* are realized." (Massumi 2002, 215, 237) In this respect, *sinthôme* is a trace that both emits from, and leads to, a void "cavity" or fissure in the real, a unique wholly feminized state of "*co-poïesis,*" an "inter-with matrixial *sinthôme*" (ibid. 229).

However, in the context of transcendental reversal, I argue that the *sinthôme* is a trace that emerges from the occurrence of a *silent singularity*. As we study the character of Didion's Maria Wyeth, the mutable imaginary trace of the *sinthôme* dynamically forms flows of meaning that originated in the imperceptible moment of what I call a pseudo-dialectic turn. Unlike Ettinger's *sinthôme* under which transgressive psychic real-things are realized on the edge of the real, under a cotranscendental configuration the *sinthôme*

[4] I retain Ettinger's spelling of *le sinthôme*, with the diacritical *accent circonflexe*. There is some typographical variance in a number of English translations of the term.

is a dynamic force emanating from an exceptional state of ontological saturation. Like Ettinger's *sinthôme*, the coalgebraic trace of a turn from the worse is realized "on the screen of fantasy" in the imaginary registers of the symptom.

In a post-evental world, in the chaotic recasting of the transcendental of that world, we shall consider an evental trace to be *normal* if, under Badiou's configuration, it constitutes the maximum degree of appearance of the ontological singularity. "The trace of the vanished event, which is … the existence of a past inexistent, and which we write ε" lays the theoretical ground for the notion of an abstract body "… *without ascribing to this body any organic status*." (Badiou 2009a, 453)

We will call an evental trace ε *normal* if it is the prior inexistent of a situation, which under the condition of ontological saturation exhibits a maximal existential coefficient of appearance. In Badiou's notation this idea is expressed in the mathemic equation "$\mathbf{E}\varepsilon = M$" whose referent marks the onto-logical origin of an evental trace.

Care must be taken if the adjective "evental" is to assigned to the pseudo-dialectic turn. The pseudo-dialectic turn follows from the occurrence of a silent singularity. Having a minimal transcendental coefficient of appearance, this occurrence is indiscernible. In Badiou's "greater logic" the formal configuration of an event entails the existence of the exception (such as $A \in A$), whose occurrence is a site exhibiting a maximal existential degree of intensity. On the other hand, a weak singularity – a silent singularity is existentially equivalent to a weak singularity – the 'existence' of a multiple A that entails its own self-inclusion is minimal: in symbols $\mathbf{E}A \Rightarrow \mathbf{E}\varnothing_A = \mu$. Under simple negation, the non-maximal existential identity value $\neg M$, is materially equivalent to the minimal existential identity value μ. A weak singularity implies a site, but not an event (ibid. 395). Only the maximal appearance of a strong singularity constitutes an event. I shall therefore refer to the site of the occurrence of a silent singularity as the site of a pseudo-dialectic turn, or alternatively "the site of the genesis of the *sinthôme*." Note however, under the greater logic, a weak singularity cannot exist in any existential sense other than it *non-exists* as a site.

Under the greater logic the *sinthôme* – denote the *sinthôme* "σ" – would ostensibly have the minimum existential identity coefficient $\mathbf{E}\sigma = \mu$. The *sinthôme* geminates in a site to trace the *non*-existence of a silent singularity in a world whose transcendental logic has been reversed under a destructive modification summoned by the occurrence of a silent singularity. The greater logic would treat this reversal as a double negation property. Under a classical Boolean *B*-transcendental ordering of a world (such as the Boolean-valued models of Zermelo-Fraenkel set theory I discussed in Chapter One) we see "an equality between the reverse of the reverse of a degree and the degree itself." (Badiou). This is an instance of the propositional logic principle of double negation and the law of the excluded middle. In the non-classical Heyting-valued worlds of the greater logic, a site is structurally equivalent to a Grothendieck topos of open sets and covers in which the "reverse" of an open set is a closed set. This "reverse" is given by the principle of complementary difference and not by simple logical negation. If two sets A and B are disjoint, their compliment $A - B$ is A. But if A is a subset of B then the relative complement of $A - B$ is empty. In a topological space, such as a Badiouian site, if A is a subset of B, the set A is said to be closed if its complement $A - B$ is open. The atomic-base multiple(s) of a topos – all Badiouian sites are Grithendieck topoi – is a collection of open sets and covers which have different onto-logical properties than the ontological properties of normal discrete sets. Under the topological configuration of a site, the "reverse" of an open set is not a closed set. This entails the fact that the algebraic structure of a site is intuitionistic (Badiou 2009a, 167, 389, 537-539). All Heyting algebras are intuitionistic logics. For technical and idealistic reasons, intuitionistic logic (IL) does not support the definitive classical laws of the excluded middle and double negation. The tautologies $(p \vee \sim p)$, $(\sim\sim p \supset p)$ and $(\sim p \vee \sim\sim p)$ of the propositional calculus [5] are not IL-theorems. In open-set topologies, the "reversal" of an open set is usually considered to be its set-theoretical compliment, the "inside-outside" of an open set, not a derivation of simple logical negation. Badiou plays on this difference by treating the reverse of a degree of a (Heyting-valued) transcendental as being "maximally 'alien' or outside to what is given, the synthesis of what is entirely exterior to it" (ibid).

Consider both the classical Boolean-valued forcing conditions, discussed in Chapter One, and the intuitionistic algebraic (transcendental) organization of worlds under Badiou's greater logic, to be *inductive initial algebras*. Badiou's topological formalism is a variant of Grothendieck's generalization of the functor category of sheaves over topological spaces, which together an axiom for the 'stitching' of compatible multiples and a treatment of subsets as subobjects, are both expressible in terms of the categorial properties of open covers (cf. (Goldblatt 1979a, 374)). At the risk of trivializing Badiou's complex and astounding iteration of Grothendieck's principles, basically, a "transcendental functor" category synthesizes an existential category of Heyting algebras and a material ontological category of open topologies (Badiou 2009a, 277-280, 289-295) to form a conceptual description of the site of an event.

However, our radical cotranscendental "reversal" of the re-configured transcendental order of a post-evental world has little to do with the principles of double negation and the excluded middle. The notion of a pseudo-dialectic turn marks a radical departure from Badiou's onto-logical treatment of the a post-evental world. When I speak of a complete, radical reversal of the transcendental of a world under the logic of the *sinthôme*, I am really referring to the principles of final coalgebra, coinduction and bisimulation. These dually *reverse* the inductive "constructor" integrity of classical and intuitionistic

[5] I have retained Russell and Whitehead's logical notation of the propositional calculus PC (Whitehead and Russell 1910). The signs "$\vee$", "$\supset$" and "$\sim$" denote binary disjunction, material implication and monadic negation respectively. "*p*" denotes a propositional variable.

inductive algebras by inverting their inductive constructor logics with a coinductive destructor logic.

The coalgebraic and coinductive reversal of the transcendental order of a world, is one into which the *sinthôme* emerges to mark the trajectory of a coinductive mutant destructor. The *sinthôme* thrives in the ill-founded world *turned* upside down by the subtle force of the occurrence of a silent singularity. The variously equivalent ontological singularities $A \in A = \{A\} = \varnothing_A$ we find in the writings of Badiou are unequivocally non-well-founded sets which are completely derivable from the anti-foundation axioms of Aczel and others (Aczel 1988, 1993; Sangiorgi 2009a).

Worlds Closed Under the Going-On

The coinductive principles of terminal coalgebras, do not entail simple reversal of the algebraic structure of the logic of a world. Silent singularities have no actual "existence" in a world, so there is no legitimate state of representation whereby the force of the *sinthôme* and the mutability of the symptom can be understood. This lead Lacan to speculate that the Borromean character of *le sinthôme* is unanalyzable. However, the *sinthôme* entails the occurrence of the cotranscendental reversal of the transcendental ordering of a world in which the mutable character of the symptom can be stabilized with a name. An external agent cannot perceive the *sinthôme* because it does not exist in representable space. The *sinthôme* is therefore unanalyzable, as Lacan has already observed. Because the *sinthôme* is an unrepresentable trace, and thereby lies outside the referential landscape of natural language, a subject can only observe the subjectivizable affect of an arbitrary name iterated at any time and over any number of iterations. When a name "sticks," the iteration ceases.

* * *

The logic of the *sinthôme*, in its most primitive form, is a coalgebraic version of the going-on. I shall define this operation as the iterative function **go-on**. The going-on is consistent with Badiou's ontology of a site, but it undergoes a radical logical inversion when the existence of the ontologically impossible – the minimal appearance of a silent singularity – turns the algebraic transcendental of a world into its coalgebraic dual. An important consideration to note, is the structural reversal of the initial algebraic transcendental ordering of a world into a cotranscendental one. As we shall see, the subject of such a world, theoretically "appears" in that world in the form of a coinductive mutant automaton whose most primitive operation is the function **go-on**.

In his essay "The Writing of the Generic" (Badiou 2003, 1-36), Badiou considers the menacingly complex, Kafkaesque world in which Beckett places the protagonist subject of *The Unnamable*. Early in the essay, Badiou gives notice of his interest in four questions which focuses his reading of *The Unnamable*. These include:

1. the *place of being* in "I"'s world and the fiction of its truth
2. "the subject, which for Beckett is essentially a question of identity"
3. the question of "the event as a supplement to immobile being to be thought," is for Beckett a problem "closely related to that of the capacities of language"
4. the question of the "*existence of the Two*, or of the virtuality of the Other." If such a Two is possible, then that Two would "be in excess of solipsism?" (ibid., 4-5)

Of course, these questions restrict Badiou's discussion to the philosophical issues which are prevalent his writing: being, the subject, the event, and the ontological and existential consequences of the event.

However, my discussion of the *sinthôme* focuses on two of these issues:

(i) the subject recast as an abstract mutant automaton

(ii) the possibility that the evental Two as a singular solipsistic alterity, emerges from the site of a silent singularity as the transitional function **go-on**. This function is dynamic and mutates under the radical flow of the *sinthôme*.

A nominal subject may synthesize a body on the level of the symptom – the *oeuvre* of James Joyce is such a body. [6] The Beckettian subject cannot escape the tyranny of the *Cogito* and the solipsistic burden of the going-on. On the other hand a particular hybrid subject does emerge in the symptomatic self-obsessed character of Maria Wyeth, the protagonist of Joan Didion's novel *Play It As It Lays*.

In the flow of the *sinthôme* and the going-on a nominal body – in name only – gathers to replace actual body. Her new abstract body bears the coalgebraic formalism [7] of the turn while, to the outside world inhabited by her friends and adversaries, the psychological atmosphere of Maria's outer world approaches terminal vacuity.

The fraught logic of the going-on and the juridical solipsistic world in which Beckett's "I" finds himself, is characterized by the staunch lack of narratological closure. Non-terminal circularity is typical of Beckett's early and middle period of writing. In Beckett's later works, Badiou notes, an evental escape appears in the "truth effects" of love, which characterizes the narratological closures that are evident in Beckett's later works. The

---

[6] On June 16, 1975, in the latter part of his career and during the opening lecture of the Paris International James Joyce Symposium, Jacques Lacan proposed the idea of "*Joyce le sinthome*" as a type of unanalyzable jouissance that lies beyond language. In her reading of Lacan, Ettinger considers how the *sinthôme* binds the disparate threads of a subject's neurosis by "by-passing repression and regression at the price of approaching dangerously primary sources of *Unheimlich* – of uncanny anxiety." (Ettinger 1999) My own hybrid version of this idea serves as a motive for my discussion of the *sinthôme*, although it differs radically from Lacan and somewhat from Ettinger.

[7] In *Logics of Worlds*, Badiou describes a body is a "singular type of object suited to serve as a support for a subjective formalism, and therefore to constitute, in a world, the agent of a possible truth." The logic of a body is a subjective formalism "capable of being borne by such bodies." (Badiou 2009a, 451) The formalism born by the subjectivizable body of the turn is exactly a terminal coalgebra.

appearance of love in Beckett's writing marks "a pivotal point, *the first numericality* … [that] constitutes a passage, or authorizes the pass, from the One of solipsism … to the infinity of being and of experience." (ibid., 28) Under the sign of love, a new numericality counts the trace of the event through which the solipsistic One turns into the continuity of Two $= \{S, \{S\}\}$. Thus an ordinal continuance, which forms the *numericality* $\{\{S, \{S\}\}, \{\{S\}, \{S, \{S\}\}\}, \ldots\}$ which is a set-theoretical the count of the going-on and of the trace of pure and truthful love of the Two. However, this rather clumsy set-theoretical characterization of the trace of Two is really the morphism ***two*** : $\{\mathbf{1}\} \rightarrow \Omega$ is in **Bont**. Remember **Bont** is the functor category **Exist** $\times$ **Set**, with a singular terminal object **1**, exponentials (product objects) and the subobject "truth" classifier Ω. However, this schematization still presumes that the going-on of the evental trace of Two is discrete, linear and stable. Such a structure does not account for the dynamics and mutability of any evental fallout, which I argue, inevitably follows a pseudo-dialectic occurrence of a singularity. Badiou's dialectic optimism stabilizes a post-evental world with the onto-logical certainty of a faithful subjectivity, which is totally missing from the mutable flows of the *sinthôme*.

In the hands of the faithful subject(s) of love, or the one body of Two, the going-on is without obstruction nor occlusion. Clearly, Badiou's dialectic idealism locates the *initial* count of Two in the material synthesis of One and other, but the count nevertheless continues, *Bestward Ho*.

For Maria Wyeth, the absence of love for another adult and her failure as a mother, any unified count as Two transforms inevitably mutates into the non-amorous reckoning of automatism. There is nothing in the staunch ontological linearity of Beckett's world, which cannot be counted, except the forbidden One count as Two. This is the law of well-founded inductive going-on. The law of going-on imposes the juridical burden of compliance under which both "I" and Maria are debilitated: Beckett by the burden of the count and Maria by the "well-founded" conceit of Hollywood's successful *élite*.

Mutant Automata and the *Sinthôme*

Named Transition Systems

The idea of bisimulation lies at the heart of the coinductive schema of the *sinthôme*. I will define bisimulation as a relation over a named transition system (NTS).

Definitions:

(i) *Relation*: Given a finite set S of states $s_1, s_2, \ldots, s_{n\text{-}1} \in S$, a relation R ranging over S is a subset of the power set construct $\mathcal{P}(S \times S)$. This means that for every relation $R \subset S \times S$ there is an ordered pair $\langle s', s'' \rangle$ of states that is an element of the relation R, i.e. for any two state $s_1, s_2 \in S$ then $\langle s_1, s_2 \rangle \in R$. The convenient the inflex notation $s_1\ R\ s_2$ can be used in place of $\langle s_1, s_2 \rangle \in R$.

(ii) *The composition of relations*: The composition of the relations $R_1, R_2,$ denoted $R_1 \circ R_2$ holds if for some state pair $\langle s, s' \rangle \in R_1 \circ R_2$, then there is some state s'' such that $\langle s, s'' \rangle \in R_1$ and $\langle s'', s' \rangle \in R_2$.

(iii) A relational structure $\mathfrak{S}$ is the pair $\langle S, R \rangle$ where S is a non-empty set of states (the domain of S) and R is a relation on S.

Definition of a model of a disrupted, non-reconstituted post-evental situation: we consider the structure of a the trace of a pseudo-dialectic turn to be a localized non-hierarchical mutating *named transition system* (NTS) [8] formally defined as the ordered triple

$$\langle S, \boldsymbol{Nom}, \xrightarrow{\nu} \rangle$$

where:

(iv) "S " denotes the set of states that constitute a situation

(v) "***Nom***" denotes a "naming" *action* ranging over S [9]

(vi) $\xrightarrow{\nu}$ is a dynamic *transition relation* over S such that $\{ \xrightarrow{\nu} : \nu \in \boldsymbol{Nom}\}$.

The variables s, t range over the states of S and the function letter ν range over the names of ***Nom***. In this case, t is a ν-derivative of s. [10]

The idea of a *localized non-hierarchical namable transition state* mirrors the relational structure known to computer science as a *labeled transition state*. In computer science data domains are often called *states*, while "label" usually refers to a particular *location* in a computer program, typically a particular line of source code. In this discussion, "namable transition system" refers to a dynamic transition system, or *trace system* that

---

[8] Notice also the American spelling "namable" which drops the "e" from the normal English language "nameable." The American spelling follows the 1958 Grove Press Inc. edition of Samuel Beckett's *The Unnamable* . This spelling is used in Beckett's own translation of his 1953 novel *L'Innomable*.

Because Kriple models are a special case of labelled state transition systems, bisimulation can also be defined in modal logic. First-order quantifiable modal logic is, according to Van Benthem's theorem, closed under bisimulation. See (Benthem 1977, 1983)

[9] The naming action "***Nom***" plays to the French language title of Beckett's *L'Innomable*. Ironically, the pair ***Nom*** and $\xrightarrow{\nu}$ form a dynamic-trace naming action that may indeed allow a subject to name the unnamable. Such an occurrence is coalgebraic and therefore modally possible. Kripke type modal logics are coalgebraic.

[10] The inflex notation $s \xrightarrow{\nu} t$ is a convenient equivalent of $\langle s, t \rangle \in \xrightarrow{\nu}$.

ranges over the states of a situation. The points of a dynamic trace are 'namable' under the *naming function* ***Nom***, which which when named are nominally represented in S.

The formal model of a mutant automaton involves the naming function ***Nom***, which is initialized at the moment of the event and continues under the evental trace and the NTS. As a dynamic naming operation on states, the operation ***Nom*** does not terminate until a name fixes. The momentary character of the event remains intact, but it is extendible in real time as the mutating trace *sinthôme*. The point at which the nomination process begins, turns out to be the greatest fixed point of a bisimulation between states.

Note also that under these schemata, the appearing of the proper inexistent of such a state is the onto-logical pair $\langle \varnothing_A, M \rangle$, whose existential degree of appearance is maximal (Badiou 2009a, 117).

Given a situation S, let the triple $\langle S, \boldsymbol{Nom}, \xrightarrow{v} \rangle$ be a named transition system and let the functional [11]

$$F_{\approx} : \mathcal{P}(S \times S) \to \mathcal{P}(S \times S)$$

be an endomap of the power set of ordered pairs of states $\langle s, t \rangle \in S$.

A relationship R holds in $\langle S, \boldsymbol{Nom}, \xrightarrow{v} \rangle$ if $R \subseteq S \times S$. In this schema, states $s, t \in S$ are given by the power set of all pairs $\langle s, t \rangle$ of S which are also elements of a binary relation R**.** We can redefine R as a bisimulation between pairs of states.

The bisimulation $F_{\approx}(R)$ is the set of all ordered pairs $\langle s, t \rangle$ such that the relation $R \subseteq S \times S$ holds for the functional $F_{\approx}$ if:

(i) for every s' of the map $s \xrightarrow{\mu} s'$ there is a t' such that $t \xrightarrow{\mu} t'$ where the relation $s' \, R \, t'$ holds

[11] In this use of the term, "functional" denotes a function space, or alternatively, a space in which functions operate.

(ii) for every t' of the map $t \xrightarrow{\mu} t'$ there is a s' such that $s \xrightarrow{\mu} s'$ where the relation $s' \, R \, t'$ holds

If we call $F_{\approx}$ the functional associated to *bisimulation*, denoted $\approx$, [12] we can then say that

(iii) $\approx$ is the greatest fixed point of $F_{\approx}$ and

(iv) $\approx$ is the largest relation R such that $R \subseteq F_{\approx}(R)$. Thus $R \subseteq \approx$ with $R \subseteq F_{\approx}(R)$

If for example, we presume the existential degree of identity between two states s_1 and s_2 is maximal, denoted $s_1 = s_2$ then for every s'_1 in an existential identity relation with a state s such that $s_1 \xrightarrow{\mathbf{Id}} s'_1$ there is a s'_2, such that $s_2 \xrightarrow{\mathbf{Id}} s'_2$ and $s'_1 = s'_2$.

This definition requires a hierarchy, because the dynamic checks (e.g. naming under ***Nom***) on the pair $\langle s_1, s_2 \rangle$ must follow those on the derivative pairs as $\langle s'_1, s'_2 \rangle$. Hence the definition is non-well-founded if the state space of the derivatives reachable from $\langle s_1, s_2 \rangle$ is infinite or includes loops.

* * *

We need to consider why bisimilarity can only be approximated inductively and that such definitions can only provide a stratified account of a situation following the occurrence of a silent singularity. This implies the idea that initial algebraic accounts of the faithful traces following an event, may only be discrete *B*-transcendental approximations of an event. To explain this in detail, formal mechanisms that draw distinctions between inductive and coinductive processes will need to be considered.

---

[12] In the literature, bisimilarity relations are usually denoted with the single tilde "$\sim$". I denote bisimilarity with the double tilde "$\approx$" leaving the single tilde to denote material negation as it is in some texts, especially (Whitehead and Russell 1910). I use the symbol "$\neg$" to denote logical negation, e.g. in modern classical logic the proposition $\neg\neg p$ is materially equivalent to p.

Transition States: The formal definition of an *unnamed state transition system* (UTS) is the tuple [13] $\longrightarrow \subseteq S \times S$, where S is a set of states, and $\longrightarrow$ is a binary transition relation between paired states $\langle s, t \rangle, \langle n, m \rangle, \ldots \in S$. If $\langle s, t \rangle \in \longrightarrow$ then the pair is considered to be a state of the relation $\longrightarrow$. Using inflex notation, the relationship is denotationally rearranged as $s \longrightarrow t$, so that s is related to t under $\longrightarrow$. Where $\longrightarrow$ is a successor function over the ordinals that index the members of S under $\longrightarrow$, then the tuple $\langle S, \longrightarrow \rangle$is a partial order on S. [14]

On the other hand, a *named* state transition system is the triple $\langle S, \boldsymbol{Nom}, \longrightarrow \rangle$ where S is a set of states, ***Nom*** is the set of the named (i.e. represented) states of S and $\longrightarrow \subseteq S \times \nu \times S$ is a ternary relation that establishes *dynamic* transitions from one named state to another. For example, if p, q are states of S and a named state α *is* an element of ***Nom*,** then the triple $\langle p, \alpha, q \rangle$ is a member of the relation $\langle p, \alpha, q \rangle \in \longrightarrow$. This triple is an element of the relation $\longrightarrow$, which in inflex form is written:

$$p \xrightarrow{\alpha} q$$

This implies that there is a transition from the state p to the state q has occurred because the transition has been given the name α. This indicates that a naming process over a named transition system is dynamic and conditional on a name terminating the system dynamics.

---

[13] In mathematics, a tuple is an ordered list of elements. For example, the list of all the possible instances of arithmetical addition over two integer variables is the tuple $\langle a, b \rangle \in +$. When $a=2$ and $b=3$, then $\langle a, b \rangle \in +$ is, in inflex form, the arithmetical proposition 2 + 3. The tuple $\langle a, b \rangle$ represents every possible value and ordered combination of a and b under +.

[14] A relation that is (referred to as) a pre-order simulation of R holds, if given two states p and q in S, q *simulates* p, possibly written $p \leq q$, if there is a simulation R such that $\langle p, q \rangle \in R$. The relation $\leq$ is the largest simulation relation over a given transition system.

Finite Approximations of Bisimulation

Let S be the states of a named transition system. Recursively define a relation $\approx$ between states $s, t, \ldots \in S$ as:

(i) $\approx_0 \stackrel{def}{=} S \times S$

(ii) $s \approx_{n+1} t$, for $n \geq 0$ then

(a) for all s with $s \xrightarrow{\mu} s'$, there is a t' such that $t \xrightarrow{\mu} t'$ and $s' \approx_n t'$;

(b) the converse is also true. For all t' with $t \xrightarrow{\mu} t'$, there is s' such that $s \xrightarrow{\mu} s'$ and $s' \approx_n t'$

(iii) $\approx_\omega \stackrel{def}{=} \bigcap_{n \geq 0} \approx_n$

Note: (iii) above finitely approximates the bisimilarity $\approx$ by defining it up to the highest countable state. Generally, $\approx$ does not coincide with $\approx_\omega$.

Given a named transition system $\langle S, \boldsymbol{Nom}, \xrightarrow{\nu} \rangle$, suppose $\nu \in \boldsymbol{Nom} \subseteq S$ and let ν be a transition between states in $\boldsymbol{Nom} \subseteq S$.

(iv) Let ν^0 be a state with no transitions.

(v) Let ν^ω be a state whose only transition is non-finitary

$$\nu^\omega \xrightarrow{\nu} \nu^\omega$$

(i) For finite transitions ν^n where $n \geq 1$ let

$$\nu^n \xrightarrow{\nu} \nu^{n\text{-}1}$$

for $\nu^n < \nu^{n\text{-}1}$ iterations of ν marking an inductive trace of the discrete finite values between 1 and $n - 1$.

Thus for all finite $n < \omega$ and $s \approx_n t$, the positive (upward) induction on n formally expresses that inductive traces give only the closest approximation of the non-finitary relation $s \approx_\omega t$ between states. Thus the inductive approximation of bisimulation terminates just short of the non-finitary bisimulation $\approx_\omega$. Note, that because the "inequation" s $!\approx_\omega$ t [15] also holds, the transition

$$t \xrightarrow{\nu} \nu^\omega$$

can only be matched by some s with *one* of the finite transitions

$$s \xrightarrow{\nu} \nu^n$$

But ν^ω $!\approx$ ν^n for all n, because $n + 1$ states can only be induced from a predecessor state of n.

This expresses the idea that a post-evental trace can only be ratified in an ontological inductive universe where the full set of axioms of set theory are inadequate unless an anti-foundation axiom is admitted.

Formal Disposition of an Abstract Mutant Automaton

Mutant automata are abstract mathematical machines. The following is a formal description of mutant automata defined as dynamic next-state machines. [16]

Consider a dynamic next-state function **go-on*** : $\mathfrak{V} \rightarrow \mathfrak{V}^{\Sigma}$ where **go-on***(v) is the function

(i) **go-on**(v, –) : $\Sigma \rightarrow \mathfrak{V}$

---

[15] The negated bisimulation, often denoted "≉", for typographical reasons will be denoted "! ≈".

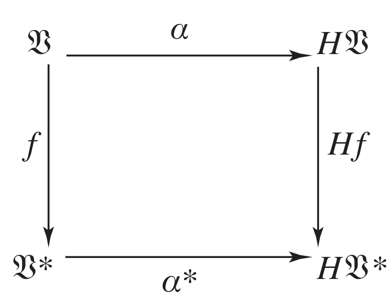
[16] The following description follows the formal mechanisms in (Adámek 2005, 158).

for every state $v \in \mathfrak{V}$ and input Σ.[17] Together, the functions **`go-on*`** and **`final`** give us the formal machinery to "construct" a deterministic automaton on a final coalgebra. This will entail taking the set of states $\mathfrak{V}$ and defining a dynamic mechanism as the function $\alpha : \mathfrak{V} \to H\mathfrak{V}$ where the functor $H\mathfrak{V}$ is $\mathfrak{V}^{\Sigma} \times \mathbf{Bool}$ (where **Bool** is the category of Boolean algebras). A deterministic automaton consists of our set of states Q and a dynamics

(ii) $\alpha : \mathfrak{V} \to H^{\Sigma}$ where $H^{\Sigma} = \mathfrak{V}^{\Sigma} \times \mathbf{Bool}$

It is important to see that there is a canonical way to make the functor $H : \mathbf{Set} \to \mathbf{Set}$ from the rule $H\mathfrak{V} = \mathfrak{V}^{\Sigma} \times \mathbf{Bool}$, (ii) above. Given a morphism $h : \mathfrak{V}_1 \to \mathfrak{V}_2$ then the functor $Hh : \mathfrak{V}^{\Sigma}{}_1 \times \mathbf{Bool} \to \mathfrak{V}^{\Sigma}{}_2 \times \mathbf{Bool}$ is the function which for every pair $\langle u, x \rangle$ with $u : \Sigma \to \mathfrak{V}_1$ and $x \in \mathbf{Bool}$ couples the composite function $h \circ u$ to x to form the pair $\langle h \circ u, x \rangle$.

Deterministic automata are examples of terminal coalgebras which can be formally specified by an endofunctor[18] H in **Set**. An endofunctor is one that reflexively maps its own category onto itself. We can formally define a coalgebra as a pair $\langle \mathfrak{V}, \alpha \rangle$ consisting of a state Q and a dynamics function $\alpha : Q \to HQ$. Homomorphisms can be defined coalgebraically. Given two coalgebras $\langle \mathfrak{V}, \alpha \rangle$ and $\langle \mathfrak{V}^*, \alpha^* \rangle$, a coalgebra *homomorphism* is a function $f : \mathfrak{V} \to \mathfrak{V}^*$ such that the categorial diagram (square)



[17] The sign "–" in schema (i) denotes an unknown state object.

[18] See fn. 3 in this chapter.

commutes.

Note that there is a canonical way of defining a categorial functor from the ontological base stuff of **Set** that includes $\mathfrak{B}$. It turns out that we are able to dynamically "unfold" a static state – so far $\mathfrak{B}$ is a static state. We can rewrite "cotranscendental of a world," as a "named transition system" in which an abstract machine, an automaton, can 'sign,' or 'name' a value in the dynamic range of the trace of some v in the system $\mathfrak{B}$.

In the algebraic semantics of **Bont** the idea of an "initial" state is central. The otological ordering of a situation is predicated on "initial" constructor principles. The initial state of constructor ontologies is the lower bound void $\varnothing$. It turns out that in the category of coalgebras, *terminal coalgebras* are an important subcategory because terminal (or final) coalgebras are the mathematical duals of the initial algebras that well-order the sort of constructor ontologies we find in **Bont**.

Schematically, a terminal coalgebra is a coalgebra $\tau : T \rightarrow HT$ which has a *unique* homomorphism $[\![\alpha]\!] : \mathfrak{B} \rightarrow T$ for every coalgebra $\alpha : \mathfrak{B} \rightarrow H\mathfrak{B}$.

At this point an automaton design should consider the role a language may play in assigning names to points in the dynamic trace *sinthôme*. Defining such a formal automata-friendly language should consider the functor

$$H = (-)^{\Sigma} \times \mathbf{Bool}$$

Here $T = \mathcal{P}\Sigma^*$ is the set of formal languages as follows: for a language L in the power set $L \in \mathcal{P}\Sigma^*$ and a state $s \in \Sigma$ let the function be

$$\texttt{go-on}(L, s) = \{w \in \Sigma^*; sw \in L\}$$

and $\texttt{final}\ (L) =$ true iff L contains some ε.

An automaton of the sort we are interested in may be formally described as a pairing $\langle \mathfrak{V}, \alpha \rangle$ of a set of states $\mathfrak{V}$ and a dynamic function α. Given this description of an automaton, then to every state v of V the unique homomorphism

$$[\![\alpha]\!] : \mathfrak{V} \rightarrow T = \exp\Sigma^*$$

assigns the language $[\![\alpha]\!](v) \subseteq \Sigma^*$ which an automaton 'introjects' (or assigns to itself) when v is an initial state. In this way the idea of an initial state is settled regarding deterministic automata. This gives us a formal language with which to self-reflexively name a state of representation.

Power Representations of Automata

States of representation can themselves be represented as coalgebras of the named transition systems that are automata. These self-representing automata can be formally described as the power set functor $\mathcal{P} : \mathbf{Set} \rightarrow \mathbf{Set}$ on a state set $\mathfrak{V}$ with the transitions

$$v \xrightarrow{s} v' \text{ for } v, v' \in \mathfrak{V} \text{ and } s \in \Sigma$$

where Σ is an set of possible choices (or decisions). This brings an aleatory character to an automaton.

For every action s, a binary relation $\xrightarrow{s}$ can be given on $\mathfrak{V}$. This can be described as the functor

$$H = \mathcal{P}(\Sigma \times -)$$

where $\mathcal{P} : \mathbf{Set} \rightarrow \mathbf{Set}$ is the power set functor. We have seen that for every state $\mathfrak{V}$, the power of representation of $\mathfrak{V}$ is given by the power set $\mathcal{P}(\mathfrak{V})$ of all the subsets of $\mathfrak{V}$. For every function $f : \mathfrak{V} \rightarrow \mathfrak{V}'$ the power function $\mathcal{P}f$ maps the subsets $M \subseteq \mathfrak{V}$ to a dynamics function $f[M] \subseteq \mathfrak{V}'$. We can correspondingly define a dynamics function

$$\mathfrak{V} \to \mathcal{P}(\Sigma \times \mathfrak{V})$$

by assigning to every state q the set $\alpha(q)$ of all the pairs $\langle s, v' \rangle \in \Sigma \times \mathfrak{V}$ with $v \xrightarrow{s} v'$.

The coalgebra homomorphisms $f: \langle \mathfrak{V}, \alpha \rangle \to \langle \mathfrak{V}', \alpha' \rangle$ are the functions which preserve the power of representation of such transitions. This means that

(i) $v \xrightarrow{s} v'$ in $\mathfrak{V}$ implies $f(v) \to f(v')$ in $\mathfrak{V}'$

and

(ii) $f(v) \xrightarrow{s} v'$ in $\mathfrak{V}'$ implies $v \xrightarrow{s} v^*$ in $\mathfrak{V}$ for some $v^* \in \mathfrak{V}$ with $f(v^*) = v'$.

Note: These homomorphisms are simple formulations of what we shall refer to as *strong bisimulations*.

It is also important to note that not every endofunctor of **Set** has a terminal coalgebra. For example the endofunctor with a power set function

$$H = \mathcal{P}(\Sigma \times -)$$

is an endofunctor of **Set**, but it does not have a terminal coalgebra. However, every endofunctor H of **Set** has a terminal coalgebra in **Class**. This is an important observation because **Class** can be considered to be the proper class of singularities. The capacity to include **Class** objects in normal situations is ontologically impossible, except in the moment of the event. Because the objects of **Class** are indeed evental singularities, there is an endofunctor (or self-representing function) in **Set** that has a terminal coalgebra in **Class**. Such terminal coalgebras define our mutant automata.

Note also that a functor is an operation on categories that preserve all relationships and morphisms of that category. We can then define the functor H : **Set**$\to$ **Set** from of the rule $H\mathfrak{V} = \mathfrak{V}^{\Sigma} \times \mathbf{Bool}$ for all states $\mathfrak{V}$.

Formal Configuration of a Mutant Automaton

I have described the existential category **Exist** as the category of complete Heyting algebras (cHA). Such algebras are generated by a collection of *constructor* operations, such as those which inductively construct the sequence of natural numbers from the void object and the application of initial principles. Such structures are often referred to as initial algebras. It is difficult to describe dynamic states, such as those occurring in computing, with initial algebras. However, dynamic structures can be described as coalgebras. Coalgebras are the structural duals of algebras. Coalgebras are not formed by initial and inductive operations, but apply instead the principle of coinduction using "destructor" or "mutation" operations. The underlying duality between algebras and induction and coalgebras and coinduction, can easily be described in category theory. This is especially evident in the category of functors with product objects described as the initial pullback (fibered product) $X \rightarrow X \times Y \leftarrow Y$ and disjoint sum objects described as the terminal pushout (fibered co-product) $X \leftarrow X + Y \rightarrow Y$ in the category **Set**.

We have also seen how the product and the co-product objects are the initial and final objects in the functor categories $\boldsymbol{F}\uparrow$: **Exist**$\rightarrow$ **Set**$\times$**Exist** and $\boldsymbol{F}\downarrow$: **Set** + **Exist**$\rightarrow$ **Coexist**.

We have seen how mutant automata can be described as abstract mathematical machines. Specifically, the formal schema of the subject of the *sinthôme*, i.e. an abstract mutant automaton, will be described as a functor object in the cotranscendental category **Coexist** in contrast to the subjectivizable body-object in the transcendental category **Exist**. The formal integrity of the following description is due to (Jacobs 1997, 15, 22), although it appears here in an abbreviated form. Any intertextual slippage should be evident.

Let T be a functor. An algebra of T is a pair consisting of the set U (a state space) and the function $a : T(U)$. Consider the state space U to be the carrier of the algebra and call the function a the operation of the algebra. An example of a basic algebraic *operation* is $\mathbf{0}$ and the successor functions $\mathbf{0} : \mathbf{1} \to \mathbb{N}$, $S : \mathbb{N} \to \mathbb{N}$ on the natural numbers $\mathbb{N}$, forming an algebra $[\mathbf{0}, \mathbf{1}] : \mathbf{1} + \mathbb{N} \to \mathbb{N}$ with the corresponding functor $T(U) \to U$ passing to the carrier set U.

For a functor T, a coalgebra is a pair $\langle U, \beta \rangle$ consisting of the set U with the $\beta : U \to T(U)$. This reverses the 'algebraic' functor $T(U) \to U$. This means that the functor $T(U) \to U$ is transcendental while the functor $U \to T(U)$ is cotranscendental.

Consider the state space U to be the carrier and the function β to be of the structure of the coalgebra $\langle U, \beta \rangle$. The difference between the algebra $T(U) \to U$ and a coalgebra $U \to T(U)$ is essentially the difference between construction and observation. An algebra consists of a carrier set U and a function $T(U) \to U$ into U, that *construct* the elements of U. Dually, a coalgebra consists of a carrier set U with a function $U \to T(U)$ passing from U, but this indicate how to 'construct' elements of U only how to operate on U because an external agent only has limited access to U.

I have argued that the *sinthôme* is the mutable trace of a subjectivizable real-valued flow emerging from the occurrence of a silent singularity. The logic of the *sinthôme*, which is one of radical existential upheaval, may be schematically expressed as a *dynamic named transition state*. The mutability of the *sinthôme* can be stabilized, point by point, in the dense abstract geometry of the open sets and covers that make up the topology of a site. By folding the action of a subjectivizable mutant non-deterministic automaton into the *sinthôme*, a naming function emerges which is exactly a namable transition system. This may be understood as an abstract desiring machine whose operational domain is a terminal coalgebra. The most basic form of such an abstract machine can be thought of as a general transition function on a non-deterministic automaton with a "linguistic" set A given as an input alphabet. The names $\alpha \in A$ are observable in a world, inasmuch as the meanings of the names are understandable in a pre-site situation. However, names must still retain some recognizable semantic currency in a world that had been turned. An observing subject necessarily exists outside the namable transition system we call *sinthôme*. The *sinthôme* is an imaginary construct. This is because any constructible system of representation that existed before the turn, on longer holds currency. A subjectivizable body is always *other* than the *sinthôme*. Although they bears semantic prodigy, the names of indiscernible states and the subjectivizable mutant automaton I am calling *subject*, are necessarily indiscernible alterities, but with nominal hooks to the actual world: *names*.

Algebras are naturally constructed from known elements, to which coalgebras only have limited access. However, coalgebras have other powerful properties. Coalgebras can effectively describe mutable and dynamic systems. This means that any notion of a world whose existential (transcendental) order is an initial algebra – such as a B-transcendental order of a situation and the transcendental order of a world – can be "deconstructed" as terminal coalgebras. This will involve the a modality of *observation* functions ranging over a dynamic orders. The simplest form of which, is a mutant automaton whose going-on is a coalgebra with a functor $T(X) = A \times X$.

Consider the functor $T(X) = A \times X$, where A denotes a multiple and X denotes an unknown state. A coalgebra has two functions $\texttt{two} : U \to A$ and $\texttt{other} : U \to U$ allowing two possible options to be given to a state $u \in U$:

(i) the operation $\texttt{two}(u)$ produces a state in U

(ii) the operation $\texttt{other}(u)$, produces the next state in U

These operations can be repeated to form another state $\texttt{two}(\texttt{other}(u)) \in A$. For each state $u \in U$ we can produce an infinite sequence $(a_1, a_2, \ldots)A^{\mathbb{N}}$ of states $a_i =$

two(**other**$^{(n)}(u) \in A$ can be produced. The sequence of elements of u what we can be observed (not constructed from) about the elements of A. of However, this gives rise to the notion that u_1 and u_2 are observationally indistinguishable without actually being equal as elements. Alternatively we can say that the elements u_1 and u_2 are *bisimilar* without being ontologically (or extensionally) identical.

Let the functor $T(X) = 1 + A \times X$ have a coalgebra **Mother** : $U_1 + A \times U$ where **Mother** stands for some *possible other* condition (it is more accurately described as possibly next) condition. [19] The notion of accessing some possible other state, or alterity, is a dynamic modal operation. Such an operation is an important factor in the construction of any desiring machine. Given a state $u \in U$ then the following conditions will hold:

(iii) either **Mother**$(u) = \varkappa(*) \in 1 = A \times U$ is in the left component of +. If so then the going-on will stop, since there is no state (element of U) with which to continue.

(iv) or, **Mother**$(u) = \varkappa'(a, u) \in 1 + A \times X$ (where A is a fixed state space) is in the right component of the disjoint sum $+$, which gives a state $a \in A$ and a next state $u \in U$ of the base carrier with which to proceed.

Repeating this procedure, an external agent can observe that for a state $u \in U$ is either a finite sequence $(a_1, a_2, \dots, a_n) \in A^*$ or it is an infinite sequence $(a_1, a_2, \dots, a_n) \in A^{\mathbb{N}}$. The observable outcomes are elements of the fixed state space $A\infty = A^* = A^{\mathbb{N}}$ of finite and infinite parts of the fixed state space A.

[19] "**Mother**" involves a linguistic play with the term "other" and the sign for the "possibly true" operator in modal logic "M." In modal logic the proposition M*p* asserts that the proposition *p* is possibly true whereas L*p* asserts that *p* is necessarily true. The Lacanian play on "**Mother**" should be evident.

A modal notion of the *sinthôme*, which in this context, is the binary desiring machine defined

> $\mathbf{go\text{-}on} : X \to \{*\} \cup X$ (where * denotes a symbol (letter, word, mark) not occurring in X. This is a simple choice machine. I choose to go on.

and

> $\langle \mathbf{two}, \mathbf{other} \rangle : X \to A \times X$ on the state space X (coalgebra function). This is a dynamic choice machine.

Now, consider the sentences "*I can't go on*" and "*I'll go on*" (Beckett 1958b, 179), and abbreviate them "**icgo**" and "**igo**" respectively. Now, let these terms denote the dynamic modal function on for a fixed set A and the functor $T(X) = A \times X$

$$\langle \mathbf{icgo}, \mathbf{igo} \rangle : A^{\mathbb{N}} \to A^* + A^{\mathbb{N}}$$

is given by the the functions $\mathbf{icgo}(\alpha) = (0)$ and $\mathbf{igo}(\alpha) = \lambda x.\, \alpha(\mathrm{x} + 1)$ (where λx is normal lambda operation on constant x).

Since **icgo** takes the first state of an infinite sequence $(\alpha(0), \alpha(1), \alpha(2), \ldots)$ A and takes the remaining it turns out that the pair of functions $\langle \mathbf{icgo}, \mathbf{igo} \rangle : A^{\mathbb{N}} \to A^* + A^{\mathbb{N}}$ is an isomorphism.

Thus for an arbitrary coalgebra $\langle \mathbf{icgo}, \mathbf{igo} \rangle : U \to A \times U$, there is a unique homomorphisms of algebras given for $u \in U$ and $n \in N$ by

$$f(u)(n) = \mathbf{two}(\mathbf{other}^{(n)}(u))$$

Thus $\mathbf{icgo} \circ f = \mathbf{two}$ and $\mathbf{igo} \circ f = \mathbf{other}$ making f a unique map of coalgebra satisfying the two equations.

When a state $u \in U$ is an infinite set (list) of elements of A arising as $\mathsf{one}(u)$, $\mathbf{one}(\mathbf{other}(u))$, $\mathbf{one}(\mathbf{other}(\mathbf{other}(u)))$, … the observable behavior of u is precisely the outcome $f(u) \in A^{\mathbb{N}}$ at u of the unique map f to the final coalgebra. Thus, the elements of the final coalgebra give the observable behavior.

When we know that $A^{\mathbb{N}}$ is a final coalgebra we can use tis finality to define functions into $A^{\mathbb{N}}$. For example, a constant function $\mathbf{const}(a) = (a, a, a, \ldots) \in A^{\mathbb{N}}$ can be defined by coinduction. Now, define the function $\mathbf{const}(a) : 1 \to A^{\mathbb{N}}$, where the structure $1 = \{*\}$ is a singleton set. We can produce a coalgebra structure $1 \to T(1) = A \times 1$ on 1 such that the $\mathbf{const}(a)$ arises by repetition. We simply define a coalgebra as the structure $1 \to A \times 1$ as the function $* \mapsto (a, *)$ letting $\mathbf{const}(a)$arise in the finality diagram

$$\begin{array}{ccc} 1 & \xrightarrow{\mathbf{const}(a)} & A^{\mathbb{N}} \\ {\scriptstyle * \mapsto (a,*)}\downarrow & & \downarrow{\scriptstyle \langle \mathbf{icgo}, \mathbf{igo} \rangle} \\ A \times 1 & \xrightarrow[id \times \mathbf{const}(a)]{} & A \times A^{\mathbb{N}} \end{array}$$

The mutant flow of the *sinthôme* is indeed a coalgebra. Coalgebra is, without a doubt, the logic of desire and the going-on.

Copoïesis of a Mutant Automaton

```
//to maria ex m#aut=[Worm] &&&
int main() {
      //what to ask?
      string id(#1):"the name sticks";
      string id(#2):"wear it!";
      string id(#3):"go on";

      if    (enunciate:= "#1") {
      cout << "#2" << endl;
}
      else {
      enunciate << "#3"<< endl;
//void action
}
return 0;
}
```

Appendix

Formal Definitions of Open Sets and Covers

Under the Grothendieck umbrella, the logic of the site is a hybrid iteration of the functorial notion of a sheaf over a topological space. Functors play a significant mathematical role in our discussion of the final coalgebras, but an extensive formal description of sheaves over topological spaces is beyond the scope of this discussion. It will suffice to say, that the logic of the site is expressible in terms of the categorial properties of open sets and covers.

Open sets are special subsets of real numbers, which are in tern, related to open intervals. This makes the topology of a site smooth and continuous.

Topological Spaces

Let X be a multiple. A collection $\mathscr{U}$ of the parts of X is called a topology if …

(i) $X \in \mathscr{U}$ and $\varnothing \in \mathscr{U}$

(ii) If $V_a \in \mathscr{U}$ for each $a \in \Lambda$, then $\bigcup \{V_a : a \in \Lambda\} \in \mathscr{U}$

(iii) If $V_i \in \mathscr{U}$ for $i = 1, 2, \ldots, n$. then $\bigcap \{V_i : i = 1, 2, \ldots, n\} \in \mathscr{U}$

The multiples in the collection $\mathscr{U}$ are open sets. The ordered pairing of X and $\mathscr{U}$ becomes the topological space $\langle X, \mathscr{U} \rangle$.

Paraphrasing this in non-notational terms:

(iv) a collection of parts for the multiple that constitutes a situation is a topology of the multiple if it contains the void and the multiple,

(v) a collection of parts for the multiple that constitutes a situation is a topology of the multiple if it contains the void and the multiple, the union of the parts of the multiple is also in the situation and the intersection of any finite collection of parts

The collection "open set," in the context of the topological space $\langle X, \mathscr{U} \rangle$, is simply the set of multiples of the topology $\mathscr{U}$.[1] The important thing to notice here is that a topological space is always a pair of open sets that form the union and intersection of open sets under certain configurations.

In mathematics, open sets are often used to distinguish between the points and subsets of a topological space. The degree of separation of those points can be specified by a separation axiom. The collection of all open sets that make up a topological space, and together with the continuous functions from one space to another, preserve the smoothness of the topology.

The idea of continuous topology and the thought that there exists closed sets which are neither open sets nor normal sets. The idea of proximity and closeness of points in a set is not a consideration of normal sets. A closed set is simply the compliment of an open set.

Open Sets

(i) A set U of $\mathbb{R}$ is an open if either $U = \varnothing$ or if for *some* $x \in U$ there is an open interval I[2] such that $x \in I \subseteq U$

---

[1] An example of an open set on a metric space is a simple circle.

A circle on a metric space can be described as both an open and closed set of discrete points. The points (x, y) satisfying the equation $x^2 + y^2 = r^2$ inscribing the radius of the circle. Open sets are all the points (x, y) satisfying the equation $x^2 + y^2 < r^2$ which lie "inside" the perimeter of the circle r^2. The union of all pairs of points is a closed set.

[2] An open interval (a, b) on the real number line is defined: $(a, b) = \{x \in \mathbb{R} : a < x < b\}$. Similarly, the real number line closed interval $[a, b]$ is defined: $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$.

(ii) A subset U of $\mathbb{R}$ is open if and only if for *every* $x \in U$ there is a positive number ε such that if $|x - y| < \varepsilon$, then $y \in U$, where $|x - y|$ denotes the *distance* between the points x and y of U

We will call a *topology* the open subsets of the multiple that constitutes a situation. This multiple, together with the set of parts of that multiple, will be called a *topological space*.

Closed Sets

(i) Let $\langle X, \mathscr{U} \rangle$ be a topological space in a world **m**. A part (subset) $U \subseteq X$ is closed if the complement (relative difference) $X - U$ is open.

(ii) In any topological space $\langle X, \mathscr{U} \rangle$ the sets X and $\varnothing$ are both open and closed.

(iii) If a multiple $X = \{a, b, c\}$ and $\mathscr{U} = \{X, \varnothing, \{a\}, \{a, b\}\}$, then the closed parts are the complements of the sets in $\mathscr{U}$. , i.e. X, $\varnothing$, $\{b, c\}$ and $\{c\}$ are closed.

(iv) If a multiple $X = \{a, b, c\}$ and $\mathscr{U} = \{X, \varnothing, \{a\}, \{a, b\}\}$, the set $\{a, c\}$ is neither open nor closed.

(v) In the topological space $\langle \mathbb{R}, \mathscr{U} \rangle$ any closed interval $[\mu, M]$ is closed since $\mathbb{R} - [\mu, M] = (-\infty, \mu) \cup (M, +\infty)$ is open. The half-open intervals $[\mu, M) = \{x \in \mathbb{R} : \mu \leq x < M\}$ and $(\mu, M] = \{x \in \mathbb{R} : \mu < x \leq M\}$ are neither open nor closed.

(vi) “Closed” does not equal “not open”

(vii) The double negation “not-not open” does not mean “open.”

Open Covers

If a set X is a topological space with parts $A \subseteq X$

(i) A collection $\mathcal{C} = \{U_\alpha : \alpha \in \Lambda\}$ of subsets of X is a *cover* of A if

$$A \subseteq \bigcup \{U_\alpha : \alpha \in \Lambda\}$$

where L is a nonempty set of index coefficients $\{1, 2, \ldots, n\}$. When this is the case we say that $\mathcal{C}$ *covers* X, or that the sets U_α *cover* X.

(ii) If U_α is an open subset of X for each $\alpha \in \Lambda$, the collection $\mathcal{C}$ is an *open cover* of A.

Bounded Parts of a Situation

Situations That Are Bounded Above

Let U be a nonempty part of a situation S, in a world **m**, whose transcendental T is indexed to the set of real numbers $\mathbb{R}$.

(i) The set U is *bounded above* if there is an element $b \in \mathbb{R}$ for which $x \leq b$ for every $x \in U$.

(a) The element b is an *upper boun*d for U.

(b) An element $m \in \mathbb{R}$ is the *least upper bound* (l.u.b.) for U if m is an upper bound for U and m is less than any other upper bound for U.

(ii) U is bounded below if there is an element $a \in \mathbb{R}$ for which $x \geq a$ for every $x \in U$.

(a) An element a is a *lower boun*d for U.

(b) An element $n \in \mathbb{R}$ is the *greatest lower bound* (g.l.b.) for U if n is a lower bound for U and n is greater than any other lower bound for U

(iii) The set U is *bounded* if it is both bounded above and bounded below.

(iv) An element $m \in \mathbb{R}$ is the *least upper bound* (l.u.b.) for U if m is an upper bound for U and m is less than any other upper bound for U.

(v) An element $n \in \mathbb{R}$ is the *greatest lower bound* (g.l.b.) for U if n is a lower bound for U and n is greater than any other lower bound for U.

Compact Topological Spaces

If the subsets of the space $\langle \mathbb{R}, \mathcal{U} \rangle$ are both closed *and* bounded then any infinite subset $A \subseteq \mathbb{R}$ has a limit point in A. (101) If all the subsets of a space $\langle \mathbb{R}, \mathcal{U} \rangle$are both closed and bounded then we shall call $\langle \mathbb{R}, \mathcal{U} \rangle$ a *compact* topological space. This is a version of the *Heine-Borel theorem for countable covers*:

(i) Let $\mathbb{R}$ have a usual topology. A subset $A \subseteq \mathbb{R}$ is compact if and only if A is both closed and bounded.

(ii) If $\mathbb{R}$ has the usual topology, then any closed interval in $\mathbb{R}$ is a compact subset.

Similarly, we shall say that the signifying surface of a body is *compact* if and only if every *representational part* of that surface is both closed and bounded.

To see that $[\mu, M]$ is compact, we use the least upper bound property.

The initial algebra of any situation is ontologically closed under the real line open interval (a, b). That is, neither the initial limit 0 nor 1 are accessible to some point in the situation because, by definition of the open interval, any point lying between the limits 0 and 1 is never equal to the limits of the interval itself. It is usual to define a *normal* algebra On the other hand, the transcendental of a world is closed under the closed interval $[\mu, M]$. This means that

We will treat the signifying surface of a body as a *usual* topological space. Let $\langle X, \mathscr{U} \rangle$ be a usual topological space, in which $\mathscr{U}$ is a collection of open sets and X is a topology. An evental site is an *unusual* topology $\langle X, \mathscr{U} \rangle^{+}$, in which the topological base of the categories **Set** and **Top** are defined as the distinction between open and non-open sets.

Site is an object in Top such that the set $V = (0, 1) \cup \{2\}$ is not an open set. There is no interval I for which $0 \in I \subseteq U$.

Situation is an object in the set $U = [0, 1)$ is not an open set. There is no interval I for which $2 \in I \subseteq U$.

Well-Founded Situations

Well-founded situations composed of those sets which belong to the class of well-founded sets, which can be defined recursively.[3] Beginning with the empty set and by iterating the power set operation, the class of well-founded situations (**WFS**) can be defined as follows. By transfinite recursion the relation $R(a)$ for every a in the class of ordinal numbers **Ord** can be defined

Let $\varnothing$ be the ordinal 0, by definition, and let R be a inductive recursive relation such that

(i) $R(0) = 0$

(ii) $R(a + 1) = \mathcal{P}(R(a))$, i.e. the next iteration of a is a value of the power set of $R(a)$

[3] This definition follows (Kunen 1980, 95)

(iii) $R(a) = \bigcup_{b<a} R(b)$ where a is a limit ordinal

We can say that the schemata (i), (ii) and (iii) formally encapsulate Badiou's thought, that "[*v*]*oid alone founds an ordinal* … it alone founds a transitive set." In contradistinction, if a multiple a admits the singularity $\{a\}$, then "we have $a \cap \{a\} = \varnothing$. But the foundational element (the site), which is a, is non-void by hypothesis. The schema a, not being founded by the void, is thus distinct from ordinals …" (Badiou 2005, 188).[4] This implies that any set that admits a singularity is non-well-founded.

Following our schemata (i), (ii) and (iii) above we can deduce by iterative induction that any multiple or situation is a well-founded set. Ontologically well-founded situatins can be generally defined as the class

$$\mathbf{WFS} = \bigcup \{R(a) : a \in \mathbf{Ord}\}$$

Thus the class of well-founded situations is defined to be the union of all recursive iterations originating from the void set but terminating with a limit ordinal whose count is $b = \omega_{-1}$. In the schemata outlined above, the recursive ordinal construction of a situation, which as Badiou points out, opens up fissure between purely mathematical ontology and other non-ontological ideas of presentation and representation (ibid.). The power set operation on R expands the closed ordinal interval $[\varnothing, \omega] = \{\varnothing < a < \omega : a \in \mathbf{Ord}\}$ to include the ordinal count of any multiple a that is recursively constructed from the void.

A relation between two sets can be defined as being well-founded on a set without having using the power set axiom. This is in contradistinction to normal definitions of well-formed sets, which invariably require the power set axiom to be instrumental in a crucial way (Kunen 1980, 98). The idea of well-formed relations over situations where the power

---

[4] In this citation, to maintain consistency with the schemata (i), (ii) and (iii), which precede it, the italic lowercase a denoting a variable in the schemata, replaces the Greek lowercase alpha, α, in the citation.

set axiom either fails (event) or has not been shown to hold will be useful in determining how post-evental relations between states may be defined.

For example, for sets A and B the first order sentence $\forall B \subset A\ (B \neq 0 \Rightarrow \exists y \in B\ (\neg \exists z \in B\ (z\ R\ y)))$ states that the relation R is well-founded on A if and only if every non-empty set has an R-minimal element (ibid.). In this case, the element y in the relations $x\ R\ y$ is called R-*minimal* in X. If the relation R totally orders the elements of A, then R is considered to be well-founded on A if and only if R well-orders A.

Bisimulation relations may or may not be well-formed but invariably they may establish well-formed relations between states which are themselves not well-formed.

Transcendental and *B*-Transcendental Intervals

A *degree p* of the intensity of appearance under the transcendental order of a world, lays on the smooth real-line closed interval $[\mu, M] = \{p \in \mathbb{R} : \mu \leq p \leq M\}$.

The *B*-Transcendental Count

A multiple s of a situation S has a *natural* count $n \in \mathbb{N}$ closed under the B-transcendental $\{0, 1\}$ of S, laying on the discrete half-open interval $[0, 1) = \{n \in \mathbb{N} : 0 \leq n < 1\}$.

Letting $\varnothing$ denote the ordinal 0, the discrete finite *ontological* count m of a situation S lays on the closed interval $[\varnothing, \omega] = \{m \in \mathbf{Ord} : \varnothing \leq n < \omega\}$.

Letting $\varnothing$ denote the ordinal 0 and ω_0 denote the least transfinite cardinal measure of a site $\mathfrak{S}$, the discrete *evental* count m of $\mathfrak{S}$ lays on the half-open discrete interval $[\varnothing, \omega_0) = \{m \in \mathbb{N} : \varnothing < n \geq \omega_0\}$.

This means that the measure of a site is non-void (in the sense of being non-empty) and the evental count of S is a transfinite cardinal measure.

Bibliography One

Cited References

Ackerkey, C. J. 1998 “Samuel Beckett and Mathematics,” *Cuardenos de Literatura Inglesa y Norteamericana* 3, pp. 77-102

Aczel, P. 1988, *Non-well-founded sets*, Stanford, CA: Center for the Study of Language and Information (CSLI)

______ 1993, “*Final Universes of Processes*,” in: S. Brookes, M. Main, A. Melton, M. Mislove and D. Schmidt, eds., *Mathematical Foundations of Programming Semantics, 9th International Conference*, New Orleans: Springer Verlag, pp. 1-28

Adámek, J. 2005 “Introduction to Coalgebra,” *Theory and Applications of Categories* 14, pp. 157–199

Badiou, A. 1982, *Théorie du sujet*, Paris: Editions du seuil

______ 1988, *L’être et l’événement*, Paris: Editions du Seuil

______ 2003, *On Beckett*, Manchester: Clinamen Press

______ 2005, *Being and Event*, New York: Continuum

______ 2006, 2006, *Logiques des Mondes: l'être et événement, 2*, Paris: Editions du Seuil

______ 2008, *Conditions*, New York: Continuum

______ 2009a, *Logics of worlds: being and event, 2*, New York: Continuum

______ 2009b, *Theory of the Subject*, New York: Continuum

Baker, C. W. 1997*, Introduction to Topology*, Malabar, FL: Krieger

Barwise, J., R. O. Gand and Y. N. Moschvakis 1971 “The Next Admissible Set,” *The Journal of Symbolic Logic* 36, pp. 108-120

Barwise, J. and L. S., Moss 1996, *Vicious Circles*, Stanford: CSLI Publications

Beckett, S. 1958, *The Unnamable*, New York: Grove Press Inc.

Benthem, J. K. von. 1977, “Modal Correspondence Theory,” *Instituut voor Logica en Grondslagenonderzoek van de Exacte Wetenschappen*, Amsterdam: Universiteit van Amsterdam

______ 1983, *Modal Logic and Classical Logic*, Napoli: Bibliopolis

Blass, A. and A. Scedrov 1989, *Freyd's Models for the Independence of the Axiom of Choice*, Providence: American Mathematical Society

Boffa, M. 1969 “Sur la Theorie des Ensembles Sans Axiome de Fondement,” *Bulletin de la Société Mathématique de Belgique* 31, pp. 16-56

Cantor, G. 1874 “Über eine Eigenschaft des Inbegriffs aller reellen algebraischen Zahlen,” *Journal für die Reine und Angewandte Mathematik / Zeitschriftenband (1874)* 77, pp. 258-262

______ 1952, *Contributions to the Founding of the Theory of Transfinite Numbers*, New York: Dover Publications

Cîristea, C., A. Kurz, D. Pattison, L. Schröder and Y. Venema 2009, “Modal Logics are Coalgebraic,” *The Computer Journal Advanced Access*: Oxford Journals

Cohen, P. J. 1963 “The Independence of the Continuum Hypothesis,” *Proceedings of the National Academy of Sciences of the United States* 50, pp. 1143-1148

______ 1964 “The Independence of the Continuum Hypothesis, II” *Proceedings of the National Academy of Sciences of the United States* 51, pp. 105-110

______ 1966, *Set Theory and the Continuum Hypothesis*, New York: W. A. Benjamin

Didion, J. 2005, *Play It As It Lays: a novel*, New York: Farrar, Straus and Giroux

Duffy, S. 2006, *Virtual Mathematics: the logic of difference*, Manchester: Clinamen Press

Easwaran, K. 2005, “A Cheerful Introduction to Forcing and the Continuum Hypothesis,” unpublished paper, http://www.ocf.berkeley.edu/~easwaran/papers/forcing.pdf

Ettinger, B. L. 1999 “Re-in/de-fuse,” *Other Voices* 1

Evans, D. 1 1996, *An Introductory Dictionary of Lacanian Psychoanalysis*, London-New York: Routledge

Felgner, U. 1969 “Die Inklusionsrelation zwischen Universa und ein abgeschwächtes Fundierungsaxiom,” *Archiv der Mathematik* 20, pp. 561-566

______ 1971, *Models of ZF-Set Theory*, Berlin-New York: Springer-Verlag

Fernando, T. 1994 “Bisimulations and Predicate Logic,” *The Journal of Symbolic Logic* 59, pp. 924-944

Finsler, P. 1975, *Aufsätze zur Mengenlehre*, Darmstadt: Wissenschaftliche Buchgemeinschaft

Fourman, M. P., C. J. Mulvey, D. S. Scott 1979, *Applications of Sheaves: proceedings of the Research Symposium on Applications of Sheaf Theory to Logic, Algebra, and Analysis, Durham, July 9-21, 1977*, Berlin-New York: Springer-Verlag

Freyd, P. 1972 “Aspects of topoi,” *Bulletin of the Australian Mathematical Society* 7, pp. 1-76

______ 1980 “The Axiom of Choice,” *Journal of Pure and Applied Algebra* 19, pp. 103-I 125

______ 1987 “All Topoi are Localic or why Permutation Models Prevail,” *Journal of Pure and Applied Algebra* 46, pp. 49-58

Fynsk, C. 1986, *Heidegger: thought and historicity*, Ithaca: Cornell University Press

Gillespie, S. 2008, The Mathematics of Novelty: Badiou's minimalist metaphysics, Melbourne: re.press

Gödel, K. and G. W. Brown 1940, *The Consistency of the Axiom of Choice and of the Generalized Continuum-Hypothesis with the Axioms of Set Theory*, Princeton: Princeton University Press

Goldblatt, R. 1979, *Topoi: the categorial analysis of logic*, Amsterdam-New York: North-Holland Pub. Co.

______ 1984, *Topoi: the categorial analysis of logic*, Amsterdam-New York: Elsevier North-Holland

______ 1987a, *Logics of Time and Computation*, Stanford: CSLI Publications

______ 1987b, *Orthogonality and Spacetime Geometry*, New York: Springer-Verlag

______ 1993, *Mathematics of Modality*, Stanford: CSLI Publications

Heidegger, M. 2006, *Sein und Zeit*, Tübingen: Max Neimeyer Verlag

Jacobs, B. and J. Rutten 1997 "A Tutorial on (Co)Algebras and (Co)Induction," *Bulletin of the EATCS* 62, pp. 222 - 259

Kunen, 1980, *Set Theory: an introduction to independence proofs*, Amsterdam-New York: North-Holland

Lawvere, F. W. and S. H. Schanuel 1997, *Conceptual mathematics : a first introduction to categories*, Cambridge: Cambridge University Press

Lemmon, E. J. 1968, *Introduction to Axiomatic Set Theory*, London-New York: Routledge-Dover Publications

Mac Lane, S. and I. Moerdijk 1992, *Sheaves in Geometry and Logic: a first introduction to topos theory*, New York: Springer-Verlag

Martin, D., and Solovay, Robert M. 1970 "Internal Cohen Extensions," *Annals of Mathematical Logic* 2, pp. 143 - 178

Massumi, B. 2002, *A Shock to Thought: expression after Deleuze and Guattari*, London-New York: Routledge

Mirimanoff, D. 1917 "Les Antinomies de Russell et de Burali-Forti et le Ploblème Fondamental de la Theorie des Ensemble," *L'enseignment Mathematique* 19, pp. 37-52

Park, D. 1981, *Concurrency and Automata on Infinite Sequences*, Berlin: Springer-Verlag

Popescu, N. 1979, *Theory of Categories*, Bucharest: Editura Academiei

Reiger, L. 1957 "A Different Kind of Model for Set Theory," *Czechoslovak Mathematical Journal* 7, pp. 323-357

Rutten, J. M. M. and D. Turi 1992, "On the Foundations of Final Semantics: non-standard Sets, Metric Spaces, Partial Orders," *Semantics: Foundation and Applications, REX Workshop, Beekergen*, The Netherlands: Lecture Notes in Computer Science, Springer, pp. 477-530

Sangiorgi, D. 2009 "On the Origins of Bisimulation and Coinduction," *ACM Transactions on Programming Languages and Systems (TOPLAS)* 31, pp. 112-151 and 115.111-115.141

Scott, D. 1960, “A Different Kind of Model for Set Theory,” *The International Congress of Logic, Methodology and Philosophy of Science*, Stanford: Stanford University Press

______ 1961 “Measurable cardinals and constructible sets,” *Bulletin de l'Académie Polonaise des Sciences. Série des Sciences Mathématiques, Astronomiques et Physiques* 9, pp. 521-524

Whitehead, A. N. and B. Russell 1910, *Principia Mathematica*, Cambridge: Cambridge University Press

Zermelo, E. 1908 “Untersuchungen über die Grundlagen der Mengenlehre I,” *Mathematische Annalen 65*, pp. 261-281

Bibliography Two

Complete Bibliography

Abraham, U. 1983 “On Forcing Without the Continuum Hypothesis,” *The Journal of Symbolic Logic* 48, pp. 658-661

Ackerkey, C. J. 1998 “Samuel Beckett and Mathematics,” *Cuardenos de Literatura Inglesa y Norteamericana* 3, pp. 77-102

Aczel, P. 1988, *Non-Well-Founded Sets*, Stanford: Center for the Study of Language and Information (CSLI)

______ 1993, “Final Universes of Processes,” in: S. Brookes, M. Main, A. Melton, M. Mislove and D. Schmidt, eds., *Mathematical Foundations of Programming Semantics, 9th International Conference*, New Orleans: Springer Verlag, pp. 1-28

Adámek, J. 2005 “Introduction to Coalgebra,” *Theory and Applications of Categories* 14, pp. 157–199

Aoki, D. S. 2006 “Letters From Lacan,” *Paragraph* 29, pp. 1-20

Apter, A. W. and J. D. Hamkins 2003 “Exactly Controlling the Non-Supercompact Strongly Compact Cardinals,” *The Journal of Symbolic Logic* 68, pp. 669-688

Awodey, S. and C. Butz 2000 “Topological Completeness for Higher-Order Logic,” *The Journal of Symbolic Logic* 65, pp. 1168-1182

Badiou, A. 1982, *Théorie du Sujet*, Paris: Editions du seuil

______ 1988, *L’être et l’événement*, Paris: Editions du Seuil

______ 1995, Beckett: l'increvable Désir, Paris: Hachette

______ 1997, Deleuze: la clameur de l'être, Paris: Hachette

______ 2000, *Deleuze: the clamor of being*, Minneapolis: University of Minnesota Press

______ 2001, Ethics: an essay on the understanding of evil, New York: Verso

______ 2003, Saint Paul: the foundation of universalism, Stanford: Stanford University Press

______ 2003, *On Beckett*, Manchester: Clinamen Press

______ 2005, *Being and Event*, New York: Continuum

______ 2006, Logiques des Mondes: l'être et événement, 2, Paris: Editions du Seuil

______ 2008, *Number and Numbers*, Cambridge: Polity Press

______ 2008, *Conditions*, New York: Continuum

______ 2009a, Logics of Worlds: being and event, 2, London: Continuum

______ 2009b, *Theory of the Subject*, London-New York: Continuum

Badiou, A., R. Brassier and A. Toscano 2006, *Theoretical writings*, New York: Continuum

Badiou, A., L. Z. Fraser and T. Tho 2007, The Concept of Model: an introduction to the materialist epistemology of mathematics, Melbourne: re.press

Badiou, A. and C. Winter 2006, *Polemics*, New York: Verso

Baker, C. W. 1997*, Introduction to Topology*, Malabar, FL: Krieger

Barr, M. and C. Wells 1985, Toposes, Triples and Theories, New York: Springer-Verlag

Barwise, J., R. O. Gand and Y. N. Moschvakis 1971 "The Next Admissible Set," *The Journal of Symbolic Logic* 36, pp. 108-120

Barwise, J. and M. Lawrence, S 1996, *Vicious Circles*, Stanford: CSLI Publications

Baur, W. 1975 "$\aleph_0$-Categorical Modules," The Journal of Symbolic Logic 40, pp. 213-220

Beckett, S. 1958, *The Unnamable*, New York: Grove Press Inc.

Bell, J. L. 2005, "The Development of Categorical Logic," *Handbook of Philosophical Logic*: Berlin-Heidelberg-New York: Springer, 2005

Bene, C. and G. Deleuze 1978, *Sovrapposizioni*, Milano: Feltrinelli Economica

Benthem, J. K. V. 1977, "Modal Correspondence Theory," Instituut voor Logica en Grondslagenonderzoek van de Exacte Wetenschappen, Amsterdam: Universiteit van Amsterdam

______ 1983, Modal Logic and Classical Logic, Napoli: Bibliopolis

Bergson, H. and G. Deleuze 1968, *Mémoire et Vie*, Paris: Presses Universitaires de France

Blanc, G. and A. Preller 1975 "Lawvere's Basic Theory of the Category of Categories," *The Journal of Symbolic Logic* 40, pp. 14-18

Blass, A. and A. Scedrov 1989, Freyd's Models for the Independence of the Axiom of Choice, Providence: American Mathematical Society

Boffa, M. 1969 "Sur la Theorie des Ensembles Sans Axiome de Fondement," *Bulletin de la Société Mathématique de Belgique* 31, pp. 16-56

Boolos, G. S. 1975 "On Second-Order Logic," *The Journal of Philosophy* 72, pp. 509-527

______ 1984 "To Be is to be a Value of a Variable (or to be Some Values of Some Variables)," *The Journal of Philosophy* 81, pp. 430-449

______ 1985 "Nominalist Platonism," *The Philosophical Review* 94, pp. 327-344

______ 1987 "A Curious Inference," *Journal of Philosophical Logic* 16, pp. 1-12

Borceux, F. and G. M. Kelly 1987 “On Locales of Localizations,” *Journal of Pure and Applied Algebra* 46, pp. 1-34

Burris, S. and H. P. Sankappanavar 1981, *A Course in Universal Algebra*, New York: Springer-Verlag

Butler, J. 1993, Bodies That Matter: on the discursive limits of “sex,” New York: Routledge

______ 2004 “Bracha’s Eurydice,” *Theory Culture Society* 21, pp. 95-100

Cameron, J., K. Cole and C. Edwards 1983, *The subjective preference theory of value*: University of East Anglia, School of Development Studies

Cantor, G. 1874 “Über eine Eigenschaft des Inbegriffs aller reellen algebraischen Zahlen,” *Journal für die Reine und Angewandte Mathematik / Zeitschriftenband (1874)* 77, pp. 258-262

______ 1952, Contributions to the Founding of the Theory of Transfinite Numbers, New York: Dover Publications

Cantor, G. and P. E. B. Jourdain 1915, Contributions to the founding of the theory of transfinite numbers, Chicago, London,: The Open Court Publishing Company

Church, A. 1940 “A Formulation of the Simple Theory of Types,” *The Journal of Symbolic Logic* 5, pp. 56-68

Cîristea, C., A. Kurz, D. Pattison, L. Schröder and Y. Venema 2009, “Modal Logics are Coalgebraic,” *The Computer Journal Advanced Access*: Oxford Journals

Clemens, J. 2006 “Had we but worlds enough, and time, this absolute, philosopher…,” *Cosmos and History: The Journal of Natural and Social Philosophy* 2, pp. 227-310

Cohen, P. J. 1961 “A Note on Constructive Methods in Banach Algebras,” *Proceedings of the American Mathematical Society* 12, pp. 159-163

______ 1963 “The Independence of the Continuum Hypothesis,” Proceedings of the National Academy of Sciences of the United States 50, pp. 1143-1148

______ 1964 “The Independence of the Continuum Hypothesis, II,” Proceedings of the National Academy of Sciences of the United States 51, pp. 105-110

______ 1966, Set Theory and the Continuum Hypothesis, New York: W. A. Benjamin

Corcoran, J. 1980 “Categoricity,” *History and Philosophy of Logic* 1, pp. 187-207

Crabbé, M. 1991 “Stratification and Cut-Elimination,” *The Journal of Symbolic Logic* 56, pp. 213-226

Cresson, A. and G. Deleuze 1952, *David Hume, sa Vie, son Oeuvre*, Paris,: Presses Universitaires de France

Cresswell, M. J. 1972 “Intensional Logics and Logical Truth,” *Journal of Philosophical Logic* 1, pp. 2-15

Critchley, S. 2002 “Ethics, Politics and Radical Democracy - The History of a Disagreement,” *Culture Machine [online]* 4

Culik, H. 1993 “Mathematics as Metaphor: Samuel Beckett and the Esthetics of Incompleteness,” *Papers on Language & Literature* 23

Deleuze, G. 1966, *Le Bergsonisme*, Paris: Presses Universitaires de France

______ 1972, *Proust and Signs*, New York,: G. Braziller

______ 1983, *Nietzsche and Philosophy*, New York: Columbia University Press

______ 1984, Kant's Critical Philosophy : the doctrine of the faculties, Minneapolis: University of Minnesota Press

______ 1986, *Cinema*, Minneapolis: University of Minnesota

______ 1986, *Foucault*, Paris: Editions de Minuit

______ 1988, *Bergsonism*, New York: Zone Books

______ 1988, *Spinoza, Practical Philosophy*, San Francisco: City Lights Books

______ 1990, *The Logic of Sense*, New York: Columbia University Press

______ 1990, Expressionism in Philosophy: Spinoza, New York-Cambridge, MA: Zone Books

______ 1990, *Pourparlers, 1972-1990*, Paris: Editions de Minuit

______ 1991, Empiricism and Subjectivity: an essay on Hume's theory of human nature, New York: Columbia University Press

______ 1993, *Critique et Clinique*, Paris: Editions de Minuit

______ 1993, *The Fold: Leibniz and the Baroque*, Minneapolis: University of Minnesota Press

______ 1994, *Difference and Repetition*, New York: Columbia University Press

______ 1995, *Negotiations, 1972-1990*, New York: Columbia University Press

______ 1997, *Essays Critical and Clinical*, Minneapolis: University of Minnesota Press

______ 2000, *Proust and Signs: the complete text*, Minneapolis: University of Minnesota Press

______ 2004, Desert Islands and Other Texts, 1953-1974, Los Angeles: Semiotext(e)

______ 2004, *Francis Bacon: the logic of sensation*, Minneapolis: University of Minnesota Press

______ 2006, The Fold: Leibniz and the Baroque, New York: Continuum

______ 2006, *Nietzsche and Philosophy*, London-New York: Continuum

Deleuze, G. and C. V. BOUNDAS 1993, *The Deleuze Reader*, New York: Columbia University Press

Deleuze, G. and A. Boyman 2001, *Pure Immanence: essays on a life*, New York: Zone Books

Deleuze, G. and F. Guattari 1975, Kafka: pour une littérature mineure, Paris: Éditions de Minuit

______ 1976, Kafka: für eine kleine Literatur, Frankfurt am Main: Suhrkamp

______ 1976, *Rhizome: an introduction*, Paris: Éditions de Minuit

______ 1977, Anti-Oedipus: capitalism and schizophrenia, New York: Viking Press

______ 1983, *On the Line*, New York: Semiotext(e)

______ 1986, *Kafka: toward a minor literature*, Minneapolis: University of Minnesota Press

______ 1986, *Nomadology: the war machine*, New York, NY, USA: Semiotext(e)

______ 1987, A Thousand Plateaus: capitalism and schizophrenia, Minneapolis: University of Minnesota Press

______ 1988, A Thousand Plateaus: capitalism and schizophrenia, London: Athlone Press

______ 1994, *What is Philosophy?*, New York: Columbia University Press

Deleuze, G. and S. HAND 1988, *Foucault*, Minneapolis: University of Minnesota Press

Deleuze, G. and E. W. HOLLAND 1999, Deleuze and Guattari's Anti-Oedipus: introduction to schizoanalysis, London - New York: Routledge

Deleuze, G. and D. Lapoujade 2006, Two regimes of madness: texts and interviews 1975-1995, Los Angeles, CA: Semiotext(e)

Deleuze, G. and C. Parnet 1977, *Dialogues*, Paris: Flammarion

______ 1987, *Dialogues*, London: Athlone Press

______ 2002, *Dialogues II*, New York: Columbia University Press

______ 2007, *Dialogues II*, New York: Columbia University Press

Deleuze, G. and L. Sacher-Masoch 1971, Masochism; an interpretation of coldness and cruelty. Together with the entire text of Venus in furs, New York: G. Braziller

______ 1971, Sacher-Masoch: an interpretation, London: Faber

______ 1989, *Masochism*, New York-Cambridge, MA: Zone Books

Denecke, K., M. Erné and S. L. Wismath 2004, *Galois Connections and Applications*, Dordrecht - London: Kluwer Academic

Denecke, K. and H. J. E. Vogel 1989, “Category Theory and Applications,” *18th Conference on universal algebra and related topics*: Pädagogischen Hochschule Karl Liebknecht Potsdam

Denecke, K. and S. L. Wismath 2000, *Hyperidentities and Clones*, Newark: Gordon and Breach

______ 2002, Universal Algebra and Applications in Theoretical Computer Science, Boca Raton: Chapman & Hall

Derrida, J. 1984 “Of an Apocalyptic Tone Recently Adopted in Philosophy,” *Oxford Literary Review* 6, pp. 3-37

______ 1993, Aporias: dying--awaiting (one another at) the “limits of truth” (mourir--s'attendre aux “limites de la vérité”), Stanford: Stanford University Press

______ 1995, *The Gift of Death*, Chicago: University of Chicago Press

______ 1997, Politics of Friendship, New York: Verso

______ 1999, *Adieu to Emmanuel Levinas*, Stanford: Stanford University Press

______ 2001, On Cosmopolitanism and Forgiveness, London: Routledge

Derrida, J. and A. Dufourmantelle 2000, *Of Hospitality*, Stanford: Stanford University Press

Diaconescu, R. 1975 "Axiom of Choice and Complementation," *Proceedings of the American Mathematical Society* 51, pp. 176-178

Didion, J. 1970, Play It As It Lays: a novel, New York: Farrar

______ 2005, *Play It As It Lays: a novel*, New York: Farrar, Straus and Giroux

Duffy, S. 2006, *Virtual Mathematics: the logic of difference*, Manchester: Clinamen Press

Easwaran, K. 2005, "A Cheerful Introduction to Forcing and the Continuum Hypothesis," unpublished paper, http://www.ocf.berkeley.edu/~easwaran/papers/forcing.pdf

Ettinger, B. L. 1992 "Matrix and Metramorphosis," differences: A Journal of Feminist Cultural Studies 4

______ 1999 "Re-in/de-fuse," *Other Voices* 1

______ 2004 "Weaving a Woman Artist With-in the Matrixial Encounter-Event," *Theory Culture Society* 21, pp. 69-94

______ 2006 "Matrixial Trans-Subjectivity," *Theory, Culture & Society* 23, pp. 218-222

Evans, D. 1996, An Introductory Dictionary of Lacanian Psychoanalysis, London-New York: Routledge

Felgner, U. 1969 "Die Inklusionsrelation zwischen Universa und ein Abgeschwächtes Fundierungsaxiom," *Archiv der Mathematik* 20, pp. 561-566

______ 1971, *Models of ZF-Set Theory*, Berlin-New York: Springer-Verlag

Fernando, T. 1994 "Bisimulations and Predicate Logic," *The Journal of Symbolic Logic* 59, pp. 924-944

Finsler, P. 1951, Über Kurven und Flächen in allgemeinen Räumen, Basel: Birkhäuser

______ 1975, *Aufsätze zur Mengenlehre*, Darmstadt: Wissenschaftliche Buchgemeinschaft

Finsler, P., D. Booth and R. Ziegler 1996, Finsler Set Theory: platonism and circularity, translation of Paul Finsler's papers on set theory with introductory comments, Basel-Boston: Birkhäuser Verlag

Fitting, M. 2004 "First-Order Intensional Logic," *Annals of Pure and Applied Logic* 127, pp. 171-193

Foss, P. 2009, The &-files : art & text 1981-2002, Delray Beach: Whale & Star

Fourman, M. P. 1980 "Sheaf Models for Set Theory," *Journal of Pure and Applied Algebra* 19, pp. 91-101

Fourman, M. P., C. J. Mulvey, D. S. Scott 1979, Applications of Sheaves: proceedings of the Research Symposium on Applications of Sheaf Theory to Logic, Algebra, and Analysis, Durham, July 9-21, 1977, Berlin-New York: Springer-Verlag

Freyd, P. 1972 "Aspects of Topoi," Bulletin of the Australian Mathematical Society 7, pp. 1-76

______ 1980 "The Axiom of Choice," Journal of Pure and Applied Algebra 19, pp. 103-125

______ 1987 "All Topoi are Localic or why Permutation Models Prevail," *Journal of Pure and Applied Algebra* 46, pp. 49-58

Fuchs, E. 1989 "Staging the Obscene Body," *TDR* 33, pp. 33-58

Fynsk, C. 1986, *Heidegger: thought and historicity*, Ithaca: Cornell University Press

Gabby, D. M. and J. Woods 2004, *Handbook of the history of logic*, Amsterdam-Boston: Elsevier

Getzler, E. and M. M. Kapranov 1998, Higher category theory: Workshop on Higher Category Theory, March 28-30, 1997, Northwestern University, Evanston, IL, Providence: American Mathematical Society

Gibson, A. 2006, Beckett and Badiou: the pathos of intermittency, Oxford-New York: Oxford University Press

Gillespie, S. 2008, The Mathematics of Novelty: Badiou's minimalist metaphysics, Melbourne: re.press

Gödel, K. 1931 "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I," *Monatshefte für Mathematik* 38, pp. 173-198

______ 1958 "Über eine Bisher noch nicht benützte Erweiterung des finiten Standpunktes," *Dialectica* 12, pp. 280-287

______ 1962, On formally Undecidable Propositions of "Principia mathematica" and Related Systems, Edinburgh-London: Oliver and Boyd

______ 1992, On formally undecidable propositions of Principia mathematica and related systems, New York: Dover

Gödel, K. and G. W. Brown 1940, The Consistency of the Axiom of Choice and of the Generalized Continuum-Hypothesis with the Axioms of Set Theory, Princeton: Princeton University Press

Goghossian, D. 1996, "What the Sokal Hoax Ought to Teach Us ," *Times Literary Supplement*, London, pp. 14 -15

Goguen, J. A. 1968/69 "The Logic of Inexact Concepts," *Synthese,* 19, pp. 325-373

Goldblatt, R. 1975 "First-Order Definability in Modal Logic," *The Journal of Symbolic Logic* 40, pp. 35-40

______ 1979, *Topoi: the categorial analysis of logic*, Amsterdam-New York: North-Holland Pub. Co.

______ 1982, Axiomatising the Logic of Computer Programming, Berlin-New York: Springer-Verlag

______ 1984, *Topoi: the categorial analysis of logic*, Amsterdam-New York: Elsevier North-Holland

______ 1984 “On the Role of the Baire Category Theorem and Dependent Choice in the Foundations of Logic,” *The Journal of Symbolic Logic* 50, pp. 412-422

______ 1984 “An Abstract Setting for Henkin Proofs,” *Topoi* 3, pp. 37-41

______ 1987, Logics of Time and Computation, Stanford: CSLI Publications

______ 1987, Orthogonality and Spacetime Geometry, New York: Springer-Verlag

______ 1988 “Varieties of Complex Algebras,” *Annals of Pure and Applied Logic* 44, pp. 173-242

______ 1993, *Mathematics of Modality*, Stanford: CSLI Publications

______ 1998, Lectures on the Hyperreals: an introduction to nonstandard analysis, New York: Springer

______ 2006, Topoi: the categorial analysis of logic, Mineola: Dover Publications

Goldblatt, R. and I. Hodkinson 2007 “The McKinsey-Lemmon Logic is Barely Canonical,” *The Australasian Journal of Logic* 5, pp. 1-19

Goldstein, R. 2005, Incompleteness: the proof and paradox of Kurt Gödel, New York: W. W. Norton

Grayson, R. J. 1983 “Forcing in Intuitionistic Systems Without Power-Set,” *The Journal of Symbolic Logic* 48, pp. 670-682

Grossman, E., R. Salado and A. Badiou 1998, *Samuel Beckett: l'écriture et la scène*, Paris: Sedes

Guattari, F. and S. Nadaud 2006, *The Anti-Oedipus Papers*, New York-Cambridge, MA: Semiotext(e)

Günther, G. 1954 “The Seetee Mind - 4 parts,” *Startling Stories* 31

______ 1954 “Aristotelian and NON-Aristotelian LOGIC - 4 parts,” *Startling Stories* 32

______ 1955 “The Soul of a Robot - 4 parts,” *Startling Stories* 32

______ 1955 “The Thought Translator - 4 parts,” *Startling Stories* 33

Haack, S. 1996, Deviant Logic, Fuzzy Logic: beyond the formalism, Chicago: University of Chicago Press

Hallward, P. 1997 “Deleuze and the ‘World Without Others,’” *Philosophy Today* 41

______ 1997 “Gilles Deleuze and the Redemption from Interest,” *Radical Philosophy*, pp. 6-21

______ 2002, “Badiou's Politics: Equality and Justice,” Culture Machine: generating research in culture and theory: Open Humanities Press

______ 2003, *Badiou: a subject to truth*, Minneapolis: University of Minnesota Press

______ 2004, Think Again: Alain Badiou and the future of philosophy, New York: Continuum

______ 2005 “Depending on Inconsistency: Badiou's answer to the “Guiding Question of All Contemporary Philosophy,” *Polygraph* 17

Handscomb, T. 1996, “Space Invaders: black satire and the BBS,” Wellington, Berlin, Dresden

Hazen, A. P. 1997 “Relations in Monadic Third-Order Logic,” *Journal of Philosophical Logic* 26, pp. 619-628

______ 1997 “Relations in Lewis's Framework without Atoms,” *Analysis* 57, pp. 243-248

Heidegger, M. 2006, *Sein und Zeit*, Tübingen: Max Neimeyer Verlag

Hendricks, V., F. Neuhaus, S. A. Pedersen, U. Scheffler and H. Wansing eds. 2004, *First-order Logic Revisited*, Berlin: Logos Verlag

Henkin, L. 1950 “Completeness in the Theory of Types,” *The Journal of Symbolic Logic* 15, pp. 81-91

Hintikka, J. 1996, *The principles of mathematics revisited*, Cambridge-New York: Cambridge University Press

Hodes, H. 1980 “Three-Valued Logics: an introduction, a comparison of various logical lexica, and some philosophical remarks,” *Annals of Pure and Applied Logic* 43, pp. 99-145

______ 1991 “Where do Sets come from?,” *The Journal of Symbolic Logic* 56, pp. 150-175

Hughes, G. E. and M. J. Cresswell 1972, *An Introduction to Modal Logic*, London: Methuen

______ 1984, *A Companion to Modal Logic*, London-New York: Methuen

Humberstone, L. 2003 “A Strange Remark Attributed to Gödel,” *History and Philosophy of Logic* 24, pp. 39-44

Jacobs, B. and J. Rutten 1997 “A Tutorial on (Co)Algebras and (Co)Induction,” *Bulletin of the EATCS* 62, pp. 222 - 259

Jané, I. 1993 “A Critical Appraisal of Second-Order Logic,” *History and Philosophy of Logic* 14, pp. 67-86

Joubert, C. 2004, “Badiou and the Ethics of Prose: revaluing Beckett,” *Polart–poétique et politique de l’art*

Kanamori, A. 2007 “Gödel and Set Theory,” *The Bulletin of Symbolic Logic* 13, pp. 153-188

______ 2008 “Cohen and Set Theory,” *The Bulletin of Symbolic Logic* 14, pp. 351-378

Koppitz, J. and K. Denecke 2006, *M-solid Varieties of Algebras*, New York: Springer

Kunen, K. 1980, Set Theory: an introduction to independence proofs, Amsterdam-New York: North-Holland

Lacan, J. 1974-1975 “Le Sinthome” (Seminar XXII), *Ornicar?* 6-11, Paris: Ornicar

Lacan, J., J.-A. Miller and J. Hulbert 1977 “Desire and the Interpretation of Desire in Hamlet,” *Yale French Studies* 55/56, pp. 11-52

Lambeck, J. 1988 “On Some Logical Connections Between Logic and Category Theory,” *Studia Logica* 48, pp. 269-278

Lambeck, J. and P. J. Scott 1986, *Introduction to Higher Order Categorical Logic*, Cambridge-New York: Cambridge University Press

Lawvere, F. W. 1963 “Functorial Semantics of Algebraic Theories,” Proceedings of the National Academy of Sciences of the United States 50, pp. 869-872

______ 1964 “An Elementary Theory of the Category of Sets,” Proceedings of the National Academy of Sciences of the United States 52, pp. 1506-1511

______ 1969 “Adjointness in Foundations,” *Dialectica* 23, pp. 281-296

Lawvere, F. W. and S. H. Schanuel 1997, Conceptual Mathematics: a first introduction to categories, Cambridge: Cambridge University Press

Lawvere, W. F. 1965 “The Category of Categories as a Foundation for Mathematics,” *Proceedings of the Conference on Categorical Algebra*, pp. 1-20

Leinster, T. 2004, *Higher Operads, Higher Categories*, Cambridge: Cambridge University Press

Lemmon, E. J. 1968, *Introduction to Axiomatic Set Theory*, London-New York: Routledge-Dover Publications

Lemmon, E. J. and D. S. Scott 1977, An Introduction to Modal Logic: the Lemmon notes, Oxford: B. Blackwell

Looijenga, E. J. N. 2008, “Smooth Manifolds”: Universiteit Utrecht: www.math.uu.nl/people/looijeng/difftop06eng.pdf

Mac Lane, S. and I. Moerdijk 1992, Sheaves in Geometry and Logic: a first introduction to topos theory, New York: Springer-Verlag

Maddy, P. 1988 “Believing the Axioms. II,” *The Journal of Symbolic Logic* 53, pp. 736-764

______ 1988 “Believing the Axioms. I,” *The Journal of Symbolic Logic* 53, pp. 481-511

______ 1993 “Does V. Equal L?,” *The Journal of Symbolic Logic* 58, pp. 15-41

Makkai, M., “Avoiding the Axiom of Choice in General Category Theory,” *Author's collection*, Montreal: McGill University-Department of Mathematics and Statistics, p. 90

Makkai, M. and G. E. Reyes 1977, First Order Categorical Logic: model-theoretical methods in the theory of topoi and related categories, Berlin-New York: Springer-Verlag

______ 1995 “Completeness Results for Intuitionistic and Modal logic in a Categorical Setting,” *Annals of Pure and Applied Logic* 72, pp. 25-101

Martin, D. and R. M. Solovay 1970 “Internal Cohen Extensions,” *Annals of Mathematical Logic* 2, pp. 143 - 178

Massui, B. 2002, A Shock to Thought: expression after Deleuze and Guattari, London-New York: Routledge

Mirimanoff, D. 1917 “Les Antinomies de Russell et de Burali-Forti et le Ploblème Fondamental de la Theorie des Ensemble,” *L'enseignment Mathematique* 19, pp. 37-52

Moss, M. and D. S. Scott 1975, *A Bibliography of Logic Books*, Oxford: Sub-Faculty of Philosophy, University of Oxford

Müller, G. H. and D. S. Scott 1978, Higher Set Theory: proceedings, Oberwolfach, Germany, April 13-23, 1977, Berlin-New York: Springer-Verlag

Oystaeyen, F. V. 2008, Virtual Topology and Functor Geometry, Boca Raton: Chapman & Hall

Park, D. 1981, Concurrency and Automata on Infinite Sequences, Berlin: Springer-Verlag

Peacocke, C. and D. S. Scott 1978, *A Selective Bibliography of Philosophical Logic*, Oxford: Sub-faculty of Philosophy, University of Oxford

Penk, A. M. 1975 “Two Forms of the Axiom of Choice for an Elementary Topos,” *The Journal of Symbolic Logic* 40, pp. 197-212

Popescu, N. 1979, *Theory of Categories*, Bucharest: Editura Academiei

Powell, W. C. 1975 “Extending Gödel's Negative Interpretation to ZF,” *The Journal of Symbolic Logic* 40, pp. 221-229

Preller, G. B. A. 1975 “Lawvere's Lasic Theory of the Category of Categories,” *The Journal of Symbolic Logic* 40, pp. 14-18

Putnam, H. 1980 “Models and Reality,” *The Journal of Symbolic Logic* 45, pp. 464-482

Quine, W. V. 1955 “On Frege's Way Out,” *Mind, New Series* 64, pp. 145-159

______ 1956 “Quantifiers and Propositional Attitudes,” *The Journal of Philosophy* 53, pp. 177-187

______ 1963, *Set theory and its logic*, Cambridge: Harvard University Press

______ 1969, *Set theory and its logic*, Cambridge: Harvard University Press

______ 1976, *The Ways of Paradox and Other Essays*, Cambridge, Mass.: Harvard University Press

Rabin, M. 1969 “Decidability of Second-Order Theories and Automata on Infinite Trees,” *Transactions of the American Mathematical Society* 141, pp. 1-35

Rayo, A. 2008, “Towards a Trivialist Account of Mathematics”: Person website: Department of Linguistics and Philosoph-MIT

Rayo, A. and G. Uzquiano 1999 “Toward a Theory of Second-Order Consequence,” *Notre Dame Journal of Formal Logic* 40, pp. 1-11

Reiger, L. 1957 “A Different Kind of Model for Set Theory,” *Czechoslovak Mathematical Journal* 7, pp. 323-357

Resnick, D. P. and D. S. Scott 2004, *The Innovative University*, Pittsburgh: Carnegie Mellon University Press

Rickels, L. A. 1999, *The Vampire Lectures*, Minneapolis: University of Minnesota Press

______ 2001, *The Case of California*, Minneapolis: University of Minnesota Press

______ 2002, *Nazi Psychoanalysis*, Minneapolis: University of Minnesota Press

______ 2010, *I think I am: Philip K. Dick*, Minneapolis: University of Minnesota Press

Rossberg, M. 2004, “First-Order Logic, Second-Order Logic, and Completeness,” in: V. Hendricks, F, F. Neuhaus, S. A. PEDERSEN, U. Scheffler and H. Wansing, eds., *First-order Logic Revisited*, Berlin: Logos Verlag Berlin, pp. 303-321

Russell, B. 1908 “Mathematical Logic as Based on the Theory of Types,” *American Journal of Mathematics* 30, pp. 222-262

Rutten, J. M. M. and D. Turi 1992, “On the Foundations of Final Semantics: non-standard Sets, Metric Spaces, Partial Orders,” *Semantics: Foundation and Applications, REX Workshop, Beekergen*, The Netherlands: Lecture Notes in Computer Science, Springer, pp. 477-530

Sangiorgi, D. 2009 “On the Origins of Bisimulation and Coinduction,” *ACM Transactions on Programming Languages and Systems (TOPLAS)* 31, pp. 112-151 and 115.111-115.141

Sarma, S. D., M. Freedman and C. Nayak 2006 “Topological Quantum Computation,” *Physics Today* 59, p. 59

Scott, D. S. 1960, “A Different Kind of Model for Set Theory,” *The International Congress of Logic, Methodology and Philosophy of Science*, Stanford: Stanford University Press

______ 1961 “Measurable Cardinals and Constructible Sets,” Bulletin de l'Académie Polonaise des Sciences. Série des Sciences Mathématiques, Astronomiques et Physiques 9, pp. 521-524

______ 1970, *The Lattice Flow Diagrams*, Oxford: Oxford University Computing Laboratory (Programming Research Group)

______ 1976, *Data Types as Lattices*, Oxford: Oxford University Computing Laboratory, Programming Research Group

Scott, D. S. and T. J. Jech 1971, *Axiomatic Set Theory*, Providence: American Mathematical Society

Sedofsky, L. 1994, “Being By Numbers: Lauren Sedofsky talks with Alain Badiou,” *ArtForum International*, pp. 84-87

______ 2006, “Matters of Appearance: an Interview with Alain Badiou,” *ArtForum*, pp. 246-253

Shapiro, S. 1985 “Second-Order Languages and Mathematical Practice,” *The Journal of Symbolic Logic* 50, pp. 714-742

______ 1990 “Second-Order Logic, Foundations, and Rules,” *The Journal of Philosophy* 87, pp. 234-261

______ 1999 “Do Not Claim Too Much: second-order logic and first-order logic,” *Philosophica Mathematica* 7, pp. 42-64

Simons, P. M. 1985 “A Semantics for Ontology,” *Dialectica* 39, pp. 193 - 216

______ 1997 “Higher-Order Quantification and Ontological Commitment ,” *Dialectica* 51, pp. 255-271

Smith, D., W. 2003 “Mathematics and the Theory of Multiplicities: Badiou and Deleuze Revisited,” *The Southern Journal of Philosophy* XLI, pp. 411-449

Tarski, A. 2002 “On the Concept of Following Logically,” *History and Philosophy of Logic* 23, pp. 155-196

Tennant, N. 1986 “The Withering Away of Formal Semantics?,” *Mind & Language* 1, pp. 302-318

Tho, T. and A. Badiou 2007 “New Horizons in Mathematics as a Philosophical Condition: an interview with Alain Badiou,” *Parrhesia*, pp. 1-11

Thomason, S. K. 1975 “Reduction of Second-Order Logic to Modal Logic,” *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 21, pp. 107-114

______ 1984 “On Constructing Instants from Events,” *Journal of Philosophical Logic* 13, pp. 85-96

Uustalu, T. and V. Vene 2002 “Least and Greatest Fixed-Points in Intuitionistic Natural Deduction,” *Theoretical Computer Science* 272, pp. 315-339

Väänänen, J. 2001 “Second-order Logic and Foundations of Mathematics,” *The Bulletin of Symbolic Logic* 7, pp. 504-520

Vene, V. 2000, “Categorial Programming with Inductive and Coinductive Types,” *Department of Mathematics*, Tartu: University of Tartu, Estonia, p. 118

Weston, T. 1976 “Kreisel, the Continuum Hypothesis and Second Order Set Theory,” *Journal of Philosophical Logic* 5, pp. 281-298

Whitehead, A. N. 1902 “On Cardinal Numbers,” *American Journal of Mathematics* 24, pp. 367-394

Whitehead, A. N. and B. Russell 1910, *Principia Mathematica*, Cambridge: Cambridge University Press

Widder, N. 2001 “The Rights of Simulacra: Deleuze and the univocity of being,” *Continental Philosophy Review* 34, pp. 437-453

Woods, J. 2003, Paradox and Paraconsistency: conflict resolution in the abstract sciences, Cambridge-New York: Cambridge University Press

Zermelo, E. 1908 “Untersuchungen über die Grundlagen der Mengenlehre I,” Mathematische Annalen 65, pp. 261-281

Ackerley, C. J. 1998 "Samuel Beckett and Mathematics", *Cuardenos de literatura inglesa y norteamericana* 3, pp. 77-102

Aczel, P. 1988, *Non-well-founded sets*, Stanford, CA: Center for the Study of Language and Information

——— 1993, "Final Universes of Processes", in: S. Brookes, M. Main, A. Melton, M. Mislove and D. Schmidt, eds., *Mathematical Foundations of Programming Semantics, 9th International Conference*, New Orleans: Springer Verlag, pp. 1-28

Adámek, J. 2005 "Introduction to Coalgebra", *Theory and Applications of Categories* 14, pp. 157–199

Badiou, A. 1982, *Théorie du sujet*, Paris: Editions du seuil

——— 1988, *L'être et l'événement*, Paris: Editions du Seuil

——— 2003, *On Beckett*, Manchester: Clinamen Press

——— 2005, *Being and event*, London ; New York: Continuum

——— 2006, *Logiques des mondes : l'être et événement, 2*, Paris: Editions du Seuil

——— 2008, *Conditions*, London ; New York: Continuum

——— 2009a, *Logics of worlds : being and event, 2*, London: Continuum

——— 2009b, *Logics of Worlds: being and event, 2*, London: Continuum

——— 2009c, *Theory of the subject*, London ; New York: Continuum

Beckett, S. 1958a, *The Unnamable*, New York: Grove Press Inc.

——— 1958b, *The UnnamableThe Collected Works of Samuel Beckett*, New York, NY: Grove Press Inc.

Benthem, J. F. A. K. v. 1977, "Modal Correspondence Theory", *Instituut voor Logica en Grondslagenonderzoek van de Exacte Wetenschappen*, Amsterdam: Universiteit van Amsterdam

——— 1983, "Modal logic and classical logic", Napoli: Bibliopolis ; Atlantic Highlands, N.J. : Distributed in the U.S.A. by Humanities Press

Boffa, M. 1969 "Sur la theorie des ensembles sans axiome de Fondement.", *ulletin de la Société Mathématique de Belgique* 31, pp. 16-56

Cantor, G. 1874 "Über eine Eigenschaft des Inbegriffs aller reellen algebraischen Zahlen", *Journal für die Reine und Angewandte Mathematik / Zeitschriftenband (1874)* 77, pp. 258-262

——— 1952, *Contributions to the founding of the theory of transfinite numbers*, New York,: Dover Publications

Cantor, G. and P. E. B. Jourdain 1915, *Contributions to the founding of the theory of transfinite numbers*, Chicago, London,: The Open Court Publishing Company

Cîrstea, C., A. Kurz, D. Pattinson, L. Schröder and Y. Venema 2009, "Modal Logics are Coalgebraic", *The Computer Journal Advanced Access*: Oxford Journals

Cohen, P. J. 1963 "The Independence of the Continuum Hypothesis", *Proceedings of the National Academy of Sciences of the United States* 50, pp. 1143-1148

——— 1964 "The Independence of the Continuum Hypothesis, II", *Proceedings of the National Academy of Sciences of the United States* 51, pp. 105-110

——— 1966, *Set theory and the continuum hypothesis*, New York,: W. A. Benjamin

Didion, J. 1970, *Play it as it lays: a novel*, New York: Farrar

——— 2005, *Play it as it lays : a novel*, New York: Farrar, Straus and Giroux

Duffy, S. 2006, *Virtual Mathematics: the logic of difference*, Manchester: Clinamen Press

Easwaran, K. 2005a, "A Cheerful Introduction to Forcing and the Continuum Hypothesis": http://www.ocf.berkeley.edu/~easwaran/papers/forcing.pdf, pp. 1-15

——— 2005b, "A Cheerful Introduction to Forcing and the Continuum Hypothesis", pp. 1-15

Ettinger, B. L. 1999 "Re- in/de -fuse", *Other Voices* 1

Evans, D. 1996, *An introductory dictionary of Lacanian psychoanalysis*, London ; New York: Routledge

Felgner, U. 1969 "Die Inklusionsrelation zwischen Universa und ein abgeschwächtes Fundierungsaxiom", *Archiv der Mathematik* 20, pp. 561-566

——— 1971, *Models of ZF-set theory*, Berlin, New York,: Springer-Verlag

Fernando, T. 1994 "Bisimulations and Predicate Logic", *The Journal of Symbolic Logic* 59, pp. 924-944

Finsler, P. 1975, *Aufsätze zur Mengenlehre*, Darmstadt: Wissenschaftliche Buchgemeinschaft [Abt. Verl.]

Fourman, M. P., C. J. Mulvey, D. S. Scott and London Mathematical Society. 1979, *Applications of sheaves : proceedings of the Research Symposium on Applications of Sheaf Theory to Logic, Algebra, and Analysis, Durham, July 9-21, 1977*, Berlin ; New York: Springer-Verlag

Freyd, P. 1972 "Aspects of topoi", *Bulletin of the Australian Mathematical Society* 7, pp. 1-76

——— 1987 "All Topoi are Localic or why Permutation Models Prevail", *Journal of Pure and Applied Algebra* 46, pp. 49-58

Fynsk, C. 1993, *Heidegger : thought and historicity*, Ithaca: Cornell University Press

Gillespie, S. 2008, *The Mathematics of Novelty: Badiou's minimalist metaphysics*, Melbourne, Vic.: re.press

Gödel, K. and G. W. Brown 1940, *The consistency of the axiom of choice and of the generalized continuum-hypothesis with the axioms of set theory*, Princeton university press;

Goldblatt, R. 1979a, *Topoi : the categorial analysis of logic*, Amsterdam; New York NY: North-Holland Pub. Co.

——— 1979b, *Topoi: the categorial analysis of logic*, Amsterdam-New York: North-Holland Pub. Co.

——— 1987a, *Logics of time and computation*, Stanford, CA: Center for the Study of Language and Information

——— 1987b, *Orthogonality and spacetime geometry*, New York: Springer-Verlag

——— 1993, *Mathematics of modality*, Stanford, Calif.: CSLI Publications; Hemel Hempstead : International Book Distributors [distributor]

Heidegger, M. 2006, *Sein und Zeit*, Tübingen: Max Neimeyer Verlag

Jacobs, B. and J. Rutten 1997 "A Tutorial on (Co)Algebras and (Co)Induction", *Bulletin of the EATCS* 62, pp. 222 - 259

Jacobs, B. a. J. R. 1997 "A Tutorial on (Co)Algebras and (Co)Induction", *Bulletin of the EATCS* 62, pp. 222 - 259

Kunen, K. 1980, *Set theory : an introduction to independence proofs*, Amsterdam ; New York

New York: North-Holland Pub. Co. ;

sole distributors for the U.S.A. and Canada, Elsevier North-Holland

Lawvere, F. W. and S. H. Schanuel 1997, *Conceptual mathematics : a first introduction to categories*, Cambridge: Cambridge University Press

Lemmon, E. J. 1968, *Introduction to axiomatic set theory*, London,

New York,: Routledge & K. Paul;

Dover Publications

Mac Lane, S. and I. Moerdijk 1992, *Sheaves in geometry and logic : a first introduction to topos theory*, New York: Springer-Verlag

Martin, D., and Solovay, Robert M. 1970 "Internal Cohen Extensions", *Annals of Mathematical Logic* 2, pp. 143 - 178

Massumi, B. 2002, *A shock to thought : expression after Deleuze and Guattari*, London ; New York: Routledge

Mirimanoff, D. 1917 "Les antinomies de Russell et de Burali-Forti et le ploblème fondamental de la theorie des ensemble", *L'enseignment mathematique* 19, pp. 37-52

Park, D. 1981, *Concurrency and automata on infinite sequences*, Berlin: Springer-Verla

Reiger, L. 1957 "A Different Kind of Model for Set Theory", *Czechoslovak Mathematical Journal* 7, pp. 323-357

Rutten, J. M. M. a. T., Daniele. 1992, "On the foundations of final semantics: Non-standard sets, metric spaces, partial orders", in: W.-P. d. R. a. G. R. J. W. de Bakker, ed., *Semantics: Foundation and Applications, REX Workshop, Beekergen*, The Netherlands: Lecture Notes in Computer Science/Springer, pp. 477-530

Sangiorgi, D. 2009a "On the Origins of Bisimulation and Coinduction", *ACM Transactions on Programming Languages and Systems (TOPLAS)* 31, pp. 112-151; 115.111-115.141

——— 2009b "On the Origins of Bisimulation and On the Origins of Bisimulation and Coinduction", *ACM Transactions on Programming Languages and Systems (TOPLAS)* 31, pp. 112-151; 115.111-115.141

Scott, D. 1960, "A Different Kind of Model for Set Theory", *The International Congress of Logic, Methodology and Philosophy of Science*, Palo Alto, California: Stanford University

——— 1961 "Measurable cardinals and constructible sets", *Bulletin de l'Académie Polonaise des Sciences. Série des Sciences Mathématiques, Astronomiques et Physiques* 9, pp. 521-524

Whitehead, A. N. and B. Russell 1910, *Principia mathematica*, [S.l.]: Cambridge U.P.

Widder, N. 2001 "The Rights of Simulacra: Deleuze and the univocity of being", *Continental Philosophy Review* 34, pp. 437-453

Zermelo, E. 1908 "Untersuchungen über die Grundlagen der Mengenlehre I", *Mathematische Annalen* 65, pp. 261-281